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Captain Vancouver's Longitudes – 1792

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1. Introduction. Captain George Vancouver's survey of the North Pacific coast of America has been characterized as being among the most distinguished work of its kind ever done. For three summers, he and his men worked from dawn to dusk, exploring the many inlets of the coastal mountains, any one of which, according to the theoretical geographers of the time, might have provided a long-sought-for passage to the Atlantic Ocean. Vancouver returned to England in poor health,[1] but with the help of his brother John, he managed to complete his charts and most of the book describing his voyage before he died in 1798.[2] He was not popular with the British Establishment, and after his death, all of his notes and personal papers were lost, as were the logs and journals of several of his officers.

Vancouver's voyage came at an interesting time of transition in the technology for determining longitude at sea.[3] Even though he had died sixteen years earlier, John Harrison's long struggle to convince the Board of Longitude that marine chronometers were the answer was not quite over. Captain Cook had proved the worth of Harrison's invention on his second voyage to the South Pacific, and Vancouver had with him six second-generation instruments by Kendall, Arnold, and Earnshaw.[4] Yet the champion of the rival method for determining longitude from the position of the Moon, the Reverend Nevil Maskelyne, was still the Astronomer Royal, and he, with the support of the Board of Longitude, steadfastly continued to promulgate the method that he had introduced, along with the first British Nautical Almanac, more than twenty years earlier in 1767.

Vancouver's book was intended to be an instructional treatise—written by a professional for professionals, and it contains records of the performance of his chronometers, and the results of his astronomical measurements of longitude. Compared with the book's text, this technical data has received relatively little attention, an omission that this paper will attempt to redress. I have confined the analysis to longitudes determined in the first year of the survey, 1792, because Vancouver made extensive use of previous results in arriving at new ones; consequently, his errors tended to be self-perpetuating. An exception to this was

[1] Naish, John. (1992). *The Health of Vancouver and His Men. Vancouver Conference on Exploration and Discovery*, Simon Fraser University, Vancouver, British Columbia.

[2] Lamb, W.Kaye, ed. (1984). *A Voyage of Discovery to the North Pacific Ocean and Round the World 1791-1795* by George Vancouver. Hakluyt Society Edition.

[3] Many books and papers have been published on the history of navigational techniques. Recommended are: Pretze, C.L. Jr., ed. (1948). *The Evolution of Celestial Navigation.* Ideal Series 26. Hearst Magazines, N.Y.; Cotter, C.H. (1968). *A History of Nautical Astronomy*; and Taylor, E.G.R. (1956 & 1971). *The Haven-Finding Art.* Hollis & Carter, London.

[4] Davies, Alun. (1993). *Chronometers of the Explorers of the Late Eighteenth and Early Nineteenth Centuries: Vancouver's Chronometers*. In Fisher R. & Johnston H., ed. *From Maps to Metaphors.* University of British Columbia Press, pp.70–84, 305–311.

Vancouver's position determination made exclusively on-site at Nootka in the fall of that year. A detailed analysis of his longitude error there can be found in an earlier paper.[5]

2. Determining longitude. Because the Earth rotates about its axis at a near-constant rate of 15º/hour, longitude can be determined by measuring the difference between local time and time on the Greenwich meridian. Using a chronometer to do this is relatively simple; the chronometer keeps a record of Greenwich mean time (usually solar, but occasionally sidereal) which is compared with the equivalent local mean time. Navigators often establish local time at noon, but it can readily be found at other times by measuring the altitude of the Sun or any other celestial body, providing the observer's latitude and the celestial co-ordinates of the Sun and any other body involved are known. Nautical Almanacs, including those used by Vancouver, routinely provide accurate tables of co-ordinates (declination and R.A.) and the so-called equation of time, i.e. the difference between apparent (real) solar time and mean solar time which arises from the inclination and slight ellipticity of the Earth's orbit.

Determining longitude from the Moon's position is also, in principle, simple, but in practice more complicated. The position of the Moon in its orbit is determined by measuring the angular distance between it and either the Sun or a selected star. Knowing the position of the Moon, it is then possible, using tables, to estimate the apparent time at Greenwich and hence, by comparison with local apparent time, to find the longitude. The problem with this method is that the orbital rotation of the Moon is 30 times slower than the axial rotation of the Earth; consequently, position measurements have to be made with great precision. Navigators who relied on the lunar-distance method required high-quality sextants, and had to spend many hours clearing their observations of the effects of sextant error, the varying sizes of the Sun and Moon's discs, refraction, and parallax.[6]

3. Vancouver's geographic positions. Vancouver arrived off the coast of California, about 180 kilometres north of San Francisco, in April 1792. From there he made his way to the Strait of Juan de Fuca, made the first circumnavigation by Europeans of Vancouver Island, and sailed south again in the fall before returning to Hawaii for the winter. On his way up the coast from California, his latitudes were a few minutes of arc too far north, but during his circumnavigation of Vancouver Island his figures seldom differ by more than one minute (1.8 km) north or south of modern determinations. His longitudes on the other hand are all about 15 minutes (about 17.7 km, depending on latitude) too far east. Considering the enormous effort that was made to determine longitude accurately, this is a significant error.

4. Vancouver's techniques.[7] Vancouver attempted to improve the accuracy of his longitude determinations by averaging large numbers of results. To amass as many estimates of the longitude of a key location as he could, Vancouver used longitude estimates made not only

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[5] Doe, N.A. (1993). *Where was Nootka 1792? An Explanation of Captain Vancouver's Longitude Error*. Journal of the Canadian Hydrographic Association. *Lighthouse* no.47, pp.15–18.

[6] Maskelyne, Nevil, ed. (1781). *Tables Requisite to be used with the Nautical Ephemeris for finding Latitude and Longitude*. 2nd Edition. Commissioners of Longitude, London. Extracts and a commentary are in Sadler, D.H. (1968). *Man is not lost.* H.M. Stationery Office, London.

[7] This section is about computational technique rather than measurement technique. For an authoritative review of the latter see David, Andrew (1993). *Vancouver's Survey Methods and Surveys*. In *From Maps to Metaphors*, ibid, pp.51–69, 291–293, 303–305.

from on-site lunar-distance observations, but also from those made at sea. It is sometimes remarked for example, that in fixing the longitude of Port Discovery, Vancouver made 220 sets of observations at that site. This is not true, as a closer reading of his book will reveal.[8] What Vancouver did was to combine what few observations he may have made at Port Discovery with many more observations made over an extended period during his voyage from Hawaii to the Strait of Juan de Fuca, together with further observations made after he left Port Discovery on May 18 and sailed to Birch Bay.

The process of transferring an estimate of longitude from one location to another was known as *reduction.* The longitude of the second location was calculated by adding the difference in longitude between the two sites, as determined by chronometer, to the longitude of the first location, as determined by astronomical observation. Thus:

$$\lambda_c(X\,|\,A) = \lambda(A) + [\ C_c(X) - C_c(A)\ ] \qquad (1)$$

where:

$\lambda_c(X\,|\,A)$ is the longitude of place X determined from astronomical observations at place A;

$\lambda(A)$ is the longitude of place A determined from on-site astronomical observations;

$C_c(X)$ is the longitude of place X according to the chronometer;

$C_c(A)$ is the longitude of place A according to the chronometer;

and the subscript c identifies different chronometer calibration methods to be defined below.

Ideally the chronometer measurements were made sufficiently close together for there to be no difference in its rate-of-going at places X and A, but when Vancouver used observations almost 40 days old, as he did at Port Discovery, then chronometer calibration became increasingly a factor in what should have been a chronometer-independent method. This is indicated by the following analysis.

We have for the longitude of any place X determined at time t_X from the chronometer:

$$C_c(X) = G(t_X) - L(t_X) - K(t_A) - R(t_A,t_X) \times (t_X - t_A) \qquad (2)$$

where:

$C_c(X)$ is the longitude, expressed in degrees (24^h=360°, west positive);

$G(t_X)$ is the chronometer reading at time t_X;

$L(t_X)$ is the local mean time at time t_X;

$K(t_A)$ is the chronometer calibration error determined at time t_A;

$R(t_A,t_X)$ is the rate the chronometer was gaining from t_A to t_X;

t_X is the time of the observation at place X;

and t_A is the time of an astronomical observation at place A.

For the purposes of this paper we can identify two different chronometer calibrations as follows. For c = 0 the chronometer is assumed to be gaining at a fixed rate of R_0 sec/day. This rate I shall identify with the rate established by Vancouver on Tahiti (Otaheite) for the

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[8] Lamb, *A Voyage...*, p.531.

Kendall K3, namely 4.03 sec/day (given in Vancouver's book sometimes as 4"2"' and sometimes as 4"3"'/day).

$$C_0(X) = G(t_X) - L(t_X) - K(t_A) - R_0 \times (t_X - t_A) \qquad \text{for all } t_X \qquad (3)$$

For c = 1, the chronometer calibration is revised at times t_{A+1}, t_{A+2}, ... t_{A+N}, etc., and t_X, the time of the observation at X, occurs between t_{A+N} and t_{A+N+1}.

$$C_1(X) = G(t_X) - L(t_X) - K(t_{A+N}) - R(t_{A+N},t_{A+N+1}) \times (t_X - t_{A+N}) \qquad (4A)$$
$$K(t_{A+N}) = K(t_{A+N-1}) + R(t_{A+N-1},t_{A+N}) \times (t_{A+N} - t_{A+N-1}) \qquad (4B)$$

By expanding the series defined by eq.(4B) in eq.(4A); using eq.(3) to eliminate the G, L, and K terms; substituting in eq.(1) with c=1; and noting the equivalence of $C_0(A)$ and $C_1(A)$, we have:

$$\lambda_1(X\,|\,A) = \lambda(A) + [\ C_0(X) - C_0(A)\ ] - [R(t_{A+N},t_{A+N+1}) - R_0] \times (t_X - t_{A+N}) - \sum_{n=0}^{N-1}\{[R(t_{A+n},t_{A+n+1}) - R_0] \times (t_{A+n+1} - t_{A+n})\} \qquad (5)$$

Eq.(5) is Vancouver's reduction formula. The dates I shall be using in this equation are the ones Vancouver gives in his book. These are a day ahead of civil dates because he did not adjust his calendar on crossing the not-then-established international date line. For the purposes of calculating elapsed days, I shall be using relative Julian days (RJD) which start at zero on December 30 (Vancouver's 31), 1791 at 0400 PST; this is Julian Day 2375573.0.

5. Vancouver's landfall result. Before arriving off the Californian coast on April 18, 1792, Vancouver made 85 sets of lunar-distance observations at sea which he reduced to his landfall position.[9] The average value of these results was 124°18'30"W; however, Vancouver was well aware that during the Pacific crossing his chronometer had been gaining at a higher rate than he had measured earlier. If he had cared to, he could have recalculated this position based on his own revised estimate for the chronometer rate. To do this, we combine eq.(1) and eq.(5) and take account of the averaging to write:

$$\sum_{m=1}^{M}\lambda_1(X\,|\,A_m)/M = \sum_{m=1}^{M}\lambda_0(X\,|\,A_m)/M - [R(t_{A+1},t_{A+2}) - R_0] \times (t_X - t_{A+1}) - [R(t_A,t_{A+1}) - R_0] \times (t_{A+1} - \sum_{m=1}^{M}t_{Am}/M) \qquad (6)$$

where:
X is the landfall position;

A_m are M mid-ocean positions A_1, A_2, A_3, etc.;

$\sum_{m=1}^{M}\lambda_1(X\,|\,A_m)/M$ is the average value of the landfall longitude calculated with re-calibrated chronometer readings;

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[9] Lamb, *A Voyage...*, p.485.

$\sum_{m=1}^{M} \lambda_0(X \mid A_m)/M$ = 124°18'30"W, the average value of the landfall longitude calculated with the chronometer readings calibrated at the Tahitian rate;

$R(t_{A+1},t_{A+2})$ = 11.5 sec/day, the chronometer rate settled at Port Discovery for observations between April 17 (RJD:+108) and May 4 (RJD:+125);

R_0 = 4.03 sec/day, the Tahitian rate;

$t_X - t_{A+1}$ is the elapsed time between landfall and April 17 (1 day);

$R(t_A,t_{A+1})$ = 8 sec/day, the chronometer rate settled at Port Discovery for observations between March 26 (RJD:+86) and April 17 (RJD:+108);

$t_{A+1} - \sum_{m=1}^{M} t_{Am}/M$ is the average elapsed time between making an astronomical determination of longitude and April 17. Since the voyage from Hawaii to their position just off the coast of California on April 17 took 21 days, we can guess that this figure might have been around 10.5 days.

Hence:

$$\sum_{m=1}^{M} \lambda_1(X \mid A_m)/M = \sum_{m=1}^{M} \lambda_0(X \mid A_m)/M - 12'17''$$

and Vancouver's corrected figure for his landfall based on pre-landfall observations is 124° 06'13"W.

This is an acceptable result. Lamb, the modern editor of Vancouver's book, suggests that the ships were at 124°04'W based on Mudge's note that they were four leagues from the shore at latitude 39°27'N.[9] However, this figure is not compatible with the given compass bearings on the land to the north, Punta Gorda at 336°, and to the southeast, south of Punta Arena at 120°. These bearings, corrected for a compass variation of 16°E, put the ships at 39°27'N, 124°15'W, which is, as Vancouver noted, roughly six leagues (33 km) from the nearest land. Accepting 124°15'W as the true longitude puts Vancouver's corrected figure for his landfall longitude too far east, as is the case for all of his other determinations.

6. The Port Discovery reference. Vancouver's key site during his circumnavigation of Vancouver Island was Port Discovery where, using eq.(1) he reduced a large number of observations made on his voyage across the Pacific and up the North American coast and combined them with some local observations. This gave him a collection of results in the form:

$$\lambda_c(K \mid A_m) = \lambda(A_m) + [\, C_c(K) - C_c(A_m) \,] \qquad m=1, 2, ... M \qquad (7)$$

Ideally the values of $\lambda_c(K \mid A_m)$ would be identical since they refer to the same location, K. In practice they were not. Differences arose from errors in the astronomical observations, which led to incorrect values for $\lambda(A_m)$, and from deviations in the chronometer rate from the assumed R_0. While little could be done to correct the lunar-distance errors, it was possible to choose alternative chronometer rates. Chronometer rates were measured whenever possible using the time between successive noons as a reference.

When this was done, Vancouver had a set of values for the longitude of Port Discovery in the form of eq.(7) with c=1. These he combined to form a grand average:

$$\lambda(\overline{K}) = \sum_{m=1}^{M} \lambda_1(K \mid A_m)/M \qquad (8)$$

At Port Discovery, M was 220 sets which represents the results of 1,320 separate observations of the Moon's position. Statistical analyses of lunar-distance longitude determinations of the late-18th century[10] show that the standard deviation of cleared sextant-reading errors was typically about 1.9' with, if several different instruments were used, a near-zero mean. Assuming uncorrelated errors, the accuracy of the Port Discovery longitude determination ought therefore to have been about $\pm 1.9 \times 29.6/\sqrt{1320} = \pm 1.5'$. That it was not is an indication of the presence of a serious systematic error in the observations, or in the procedures used to process them.

7. Retroactive corrections. To ensure consistency, Vancouver used the Port Discovery reference to retroactively correct his earlier determinations. The equation he used was:

$$\lambda_1(X_m \mid \overline{K}) = \lambda(\overline{K}) - [\, C_1(K) - C_1(X_m) \,] \qquad (9)$$

where:

$\lambda_1(X_m \mid \overline{K})$ is the longitude of place X_m determined from the averaged results of longitude measurements made at the places A_1, A_2, A_3, etc.

Eq.(5), which gives the procedure for carrying a longitude forward from A to any X, including K, can also be used to define how longitudes were carried back from the key location K to any X. Thus by re-arranging eq.(5) we have:

$$\lambda_1(X \mid \overline{K}) = \lambda(\overline{K}) - [\, C_0(K) - C_0(X) \,] + [R(t_{K-N-1},t_{K-N}) - R_0] \times (t_{K-N} - t_X) + \sum_{n=0}^{N-1} \{[R(t_{K-n-1},t_{K-n}) - R_0] \times (t_{K-n} - t_{K-n-1})\} \qquad (10)$$

where:

t_K is the time of the observation at the key location K;

the chronometer calibration was revised at times t_{K-1}, t_{K-2}, ... t_{K-N}, etc.

t_X, the time of the observation to be corrected, occurs between t_{K-N-1} and t_{K-N}

and, as before, the equivalence $24^h = 360°$ is to be understood.

For example, for landfall on April 18, N=0, and we have:

$\lambda(\overline{K})$ = 122°37'41"W, the Port Discovery grand-average longitude;

$C_0(K)$ = 124°01'W, the Port Discovery longitude by the chronometer calibrated at the Tahitian rate;

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[10] Doe, N.A. (1994). *Comments on the accuracy of late 18th-century sextants* (unpublished paper). Also Sadler, D.H. (1976). Lunar Distance and the Nautical Almanac. *Vistas in Astronomy.* 20, pp.113-121.

$C_0(X)$ = 125°W, the landfall longitude by the chronometer calibrated at the Tahitian rate;

t_K = May 4 (RJD:+125);

t_X = April 18 (RJD:+109);

$R(t_{K-1},t_K)$ = 11.5 sec/day, the revised chronometer rate adopted for April 17 to May 4;

R_0 = 4.03 sec/day, the Tahitian rate.

The corrected landfall longitude, $\lambda_1(X|\overline{K})$, from eq.(10), was therefore 124°06'34"W. This is only 21" west of the 124°06'13"W position computed above exclusively from pre-landfall data—much too close to be a coincidence. I shall return to this shortly.

The figure given in Vancouver's book for the revised longitude of landfall is 123°35'W,[8] but this is almost certainly a miscalculation. As Lamb points out, this would place him a dozen miles inland. It is impossible to be sure what arithmetic mistake was made; however, Vancouver's 123°35'W for $\lambda_1(X|\overline{K})$ can be reproduced exactly by using in eq.(10) R_0 = 11.92 sec/day, which is the chronometer rate that Vancouver actually measured at Port Discovery, instead of the correct R_0 = 4.03 sec/day.

8. Lunar-distance and chronometer relationships. Consider now why the longitude of Vancouver's landfall derived from pre-landfall observations should be virtually identical to that derived from the chronometer relative to Port Discovery. The only tenable explanation is that the pre-landfall observations must have been retroactively "corrected" to be compatible with the Port Discovery longitude. The evidence for this is as follows.

On five occasions crossing the Pacific, Vancouver reports making a lunar-distance determination of his longitude. At the same time he records for comparative purposes his longitude by the chronometer. A typical entry in his book is:[11]

... On Sunday morning the 15th (of April), *I got one set of lunar distances, which at noon gave the longitude 232°56'30"; by the chronometer 232°7'45"; and by the dead reckoning 229°39'; ...*

RJD	Vancouver's date	Chronometer	Lunar Distance.	Formula	Formula "Error"
47	Feb 16	155°53'45"	155°54'07"	155°54'03"	–0'04"
57	Feb 26	156°20'00"	156°12'00"	156°12'00"	0'00"
89	Mar 29	146°13'30"	145°38'45"	145°38'58"	+0'13"
106	Apr 15	127°52'15"	127°03'30"	127°03'38"	+0'08"
108	Apr 17	124°42'00"	123°52'00"	123°51'43"	–0'17"

The chronometer longitudes are as reported by Vancouver at the Otaheitean rate.
Lunar distance longitudes are as reported by Vancouver.
Formula (lunar-distance) longitudes are derived from the chronometer longitudes using:

form. long. = chrono. long. – 49.7414" × (RJD – 47.352)

TABLE 1 Vancouver's Pacific longitudes compared

[11] Lamb, *A Voyage...*, p.484.

Table 1 shows these simple statements to be very deceptive. The longitudes labelled "formula" in Table 1 are derived solely from the chronometer longitudes; nevertheless, they are practically identical to the lunar-distance longitudes. The standard deviation of the difference is only 10". In other words, the lunar-distance longitudes quoted by Vancouver are not observations; the values must have been calculated.

My guess is that Vancouver is reporting, not $\lambda(A)$ as he implies, but revised figures $\lambda_1(A\,|\,\overline{k})$, i.e. he has reduced his original astronomical determinations to a key location k, and then "corrected" the original observations. If in eq.(10) a single chronometer rate R_1 is applicable, then plotting the differences between the Tahitian-rate chronometer longitudes $C_0(X)$ and the corrected longitudes $\lambda_1(X\,|\,\overline{k})$ as a function of time results in a straight line with a slope of $(R_1 - R_0)$. This is exactly what Table 1 shows, the value of R_1 being 7.35 sec/day (49.741"W/day + the Tahitian 4.03 sec/day).

Was the key location, k, Port Discovery? Almost certainly not. If it were, then it could be shown that the RJD at Port Discovery was +147, i.e. May 26. This is too late; by then Vancouver had already left Port Discovery, and the chronometer calibration factor R_1 of 7.35 sec/day is too low. It is more likely that at the key location the chronometer longitude $C_0(k)$ was equated to the averaged lunar-distance longitude $\lambda(\overline{k})$ and the date the ships were at the key location, between Tahiti and Hawaii, was about February 16, 1792 (RJD:+47.4). The full significance of this result will become apparent shortly.

9. The behaviour of the Kendall K3. After his arrival in Californian waters, Vancouver sailed north along the coast recording as he went the latitudes and longitudes of various landmarks. Lamb has conveniently included modern co-ordinates in his edition of Vancouver's book and I find no error in his work other than the true longitude of Cape Blanco (Vancouver's Orford) which is 5' west of Lamb's location. In one case, near Cape Shoalwater on April 27, Vancouver appears to have reported the position of his ship instead of that of the cape being observed. With this particular result excluded, the average error for the 14 longitude measurements between Punta Gorda and Port Discovery is 12'54"E.

These measurements are of interest because, in the absence of Vancouver's original notes, they provide the most reliable evidence of how the Kendall K3 chronometer performed during this part of the voyage. To recover the "raw" data needed to understand Vancouver's problems, we re-arrange and simplify eq.(10):

$$C_0(X) = \lambda_1(X\,|\,\overline{K}) - \lambda(\overline{K}) + C_0(K) - [R(t_{K-1},t_K) - R_0] \times (t_K - t_X) \qquad (11)$$

where:

$\lambda_1(X\,|\,\overline{K})$ is the longitude published in Vancouver's book;

$\lambda(\overline{K})$ = 122°37'41"W, Vancouver's Port Discovery longitude;

$C_0(K)$ = 124°01'W, Port Discovery by the chronometer;

$R(t_{K-1},t_K)$ = 11.5 sec/day;

R_0 = 4.03 sec/day;

t_K = May 4 (RJD:+125);

and t_X = time at place on the coast.

From eq.(11) we can recover the chronometer longitudes $C_0(X)$ at the Tahitian rate at the various known locations. These figures can be added to the chronometer longitudes for known locations in Hawaii, for which calculations are not necessary because Vancouver was already using the Tahitian calibration. We can then estimate the chronometer's behaviour during the spring of 1792 by making a polynomial regression analysis of the relationship between the true longitudes and the Tahitian-rate chronometer longitudes.

This analysis gives the following relationship, valid from March 1 (RJD:+61), just before the ships reached the Hawaiian Islands, to May 13 (RJD:+134):

$$T(X) = C_0(X) - 30'53'' - 1'15.7'' \times d - 0.6338'' \times d^2 \qquad (12)$$

where:

$T(X)$ is the true longitude of place X;

$C_0(X)$ is the Tahitian-rate chronometer longitude of place X;

d is the number of days after Apr 9 (RJD:+100) the observation was made, i.e. d = RJD – 100.

Some of the predictions that this relationship makes are:

- longitude of Port Discovery on May 4 (RJD:+125): 122°51'58"W. This is only 32" east of true
- longitude of landfall on April 18 (RJD:+109): 124°16'54"W
- longitude of Waikiki on March 8 (RJD:+68): 157°49'03"W. This is only 12" east of true
- chronometer gain on May 13 (RJD:+134): 11.97 sec/day. The measured value was 11.92 sec/day
- accumulated chronometer gain from Mar 26 to May 4 in excess of 4.03 sec/day: 214.9 sec. Vancouver's estimate was 214.3 sec.

The implied assumption in eq.(12) is that the chronometer accelerated at a constant rate. Although this is mathematically convenient, there is no reason why the chronometer should have progressed smoothly from its Tahitian rate of 4.03 sec/day to the 11.92 sec/day measured by Vancouver at Port Discovery; however, the evidence supports the conjecture that no sudden change in rate occurred.

10. Vancouver's analysis. Figure 1 provides a summary of the analysis of the chronometer characteristics. The curves show estimates of the chronometer error after correction at the Tahitian rate. The reference point for the estimates is Vancouver’s calibration at Point Venus on Tahiti; this calibration is shown as a horizontal line in Figure 1.

Vancouver’s first landfall after leaving Tahiti was Hawaii, and although Vancouver made no lunar-distance longitude determinations during his first visit there, he did compare his chronometer longitudes with those determined by Captain Cook and Lieutenant King. These are the points marked in the Figure. Vancouver justifiably took these as confirming the correctness of his Tahitian calibration.

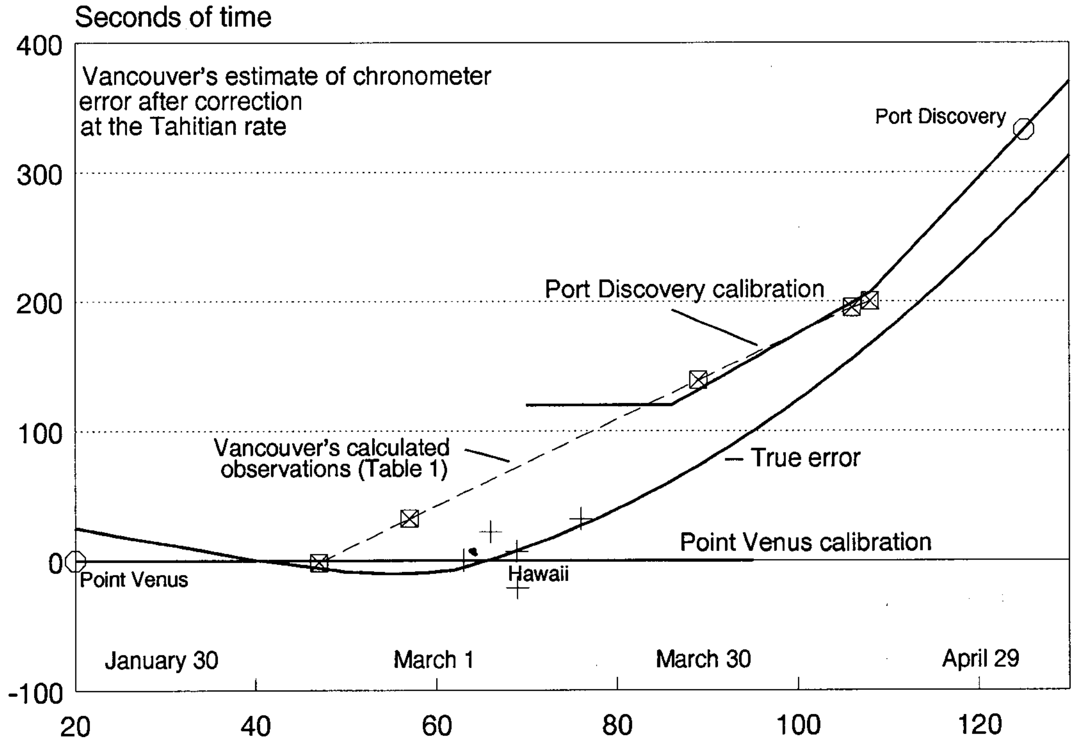

FIGURE 1   Vancouver's estimates of the K3 chronometer error and the true error.

The solid middle curve shows the author's estimate of the true error. Vancouver calibrated the chronometer on Tahiti to a longitude that was 6'15" too far west (25 sec fast). Evidently, after leaving the island, the Kendall K3 chronometer rate dropped below 4.03 sec/day for a time, as it did in New Zealand, and this eliminated the calibration error sometime in February.[12] From March 1 onwards, the curve is derived from eq.(12) and shows the time equivalent of [$C_0(X) - T(X)$].

The upper curve in Figure 1 shows Vancouver's Port Discovery calibration. On March 26 (RJD:+86), the initially-constant error starts to increase at 8 – 4.03 = 3.97 sec/day. On April 17 (RJD:+108), the rate increases to 11.5 – 4.03 = 7.47 sec/day. The curve is positioned so that at Port Discovery the accumulated error is 333 seconds which is the time-equivalent difference reported by Vancouver between his chronometer longitude (124°01'W) and his lunar-distance longitude (122°37'41"W).

While looking at Figure 1, it may help to remember that Greenwich time was obtained by subtracting any error from the indicated time; hence an over-estimated error resulted in Greenwich time being set too slow, and the conclusion was drawn that the longitude was further east than was actually the case.

[12] Doe, N.A. (1994). *Vancouver's Assessment of Kendall's Chronometer K3 in 1791/1792*. Journal of the Canadian Hydrographic Association. *Lighthouse* no.50, pp.15–26.

11. Vancouver's dilemma. Figure 1 sums up very succinctly the dilemma that Vancouver faced. His assessment of the chronometer's behaviour over the period from late March to May was good. Apart from the offset, there is no practical difference between the form of his calibration curve and that of the true error curve; yet, because of the offset, it was impossible for Vancouver to reconcile the Port Discovery calibration with observations made earlier in the year.

There were four possible solutions to his puzzle—

- the chronometer had started gaining at a substantially higher rate than he had reckoned after leaving Hawaii. This would account for its longitude being apparently too far west at Port Discovery. The difficulty with this solution was that his measurements of the rate-of-going of the chronometer gave no indication that such an increase had occurred
- Cook and King's longitudes in the Hawaiian Islands were too far west and the chronometer had slowly started gaining on the voyage from Tahiti to Hawaii. This would have made the chronometer longitudes appear to be correct in Hawaii, even though they were not. The dotted line in Figure 1 which bridges the gap between the two curves is the line that Vancouver used to "manufacture" his lunar-distance observations discussed above. Vancouver was obviously seriously considering this solution to his problem, before he made his final choice
- the chronometer had not gained more than calculated, but both the Hawaiian longitudes and the reference longitude at Point Venus were equally too far west. This too could explain why the Port Discovery chronometer longitude was too far west. The problem here was that the longitude of Point Venus on Tahiti was considered to be well established. Cook visited the island on all three of his voyages
- or the corrected chronometer longitude was substantially correct and the longitude of Port Discovery as indicated by his astronomical observations was too far east.

Vancouver's choice of solution is interesting; he chose to adopt the Port Discovery figure and acknowledge in his words that *by some observations made prior to the 26th of March, it* (the chronometer) *appeared to have deviated manifestly from the truth*. In making this decision, Vancouver was complying with instructions given by the Board of Longitude to William Gooch, the intended astronomer for the voyager. These were that he was to determine his longitude by lunar distances, but only note (set down) the longitude indicated by the chronometer.

Vancouver had, of course, made the wrong decision. Port Discovery was actually close to 122°52'30"W, and his prodigious number of astronomical measurements had placed him 14.8' east of his true position. The Hawaiian longitudes were good, the longitude at Point Venus, although indeed too far west, was not that seriously in error, and Vancouver's assessment of the chronometer rates was also good.

In the light of this discussion, Vancouver's choice of longitude for the observatory near Point Venus is interesting. The correct value is 149°29.5'W, but Vancouver quotes a figure 6.3' west of this at 149°35.8'W which he attributes to Cook. That Cook would have agreed is doubtful; Cook's chart of the island,[13] his journal,[14] and records of his observations,[15] all

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[13] Cook, James. (1769). *Chart of the Island Otaheite*. Reproduced in Wharton, W.J.L., ed. (1893). *Captain Cook's Journal during his first voyage... 1768-1771*, facing p.81.

place the observatory at 149°30'W, which is almost exactly right. It appears more likely that Vancouver's figure is derived from the 149°34.8'W reckoning of Charles Green, the astronomer on Cook's first voyage, which was endorsed by the equally-eminent astronomer William Wales in his notes made on Cook's second voyage.[16] Vancouver in his book adds further interest by giving a chronometer calibration figure at Point Venus that is not what he actually uses in his subsequent calculations, but which quite possibly was derived from Cook's correct longitude.[12]

12. Errors in the astronomical determinations. The only outstanding question now is why Vancouver's determination of the longitude of Port Discovery was so far east. The answer is that his Nautical Almanac contained a systematic, or non-random error. To show this, I used the same five-step procedure that was successfully used to explain Cook's relatively inaccurate determination of the longitude of Nootka in 1778,[17] and Vancouver's results obtained 14 years later.[6] The procedure uses a very accurate modern ephemeris computer program[18] to re-calculate the lunar distances in the Nautical Almanac for 1792.

Figure 2 shows the results of this procedure for the period March 26 (RJD:+86) to June 12 (RJD:+164). This covers the period over which lunar-distance observations were made, commencing in mid-ocean in March and finishing in Birch Bay in June where 28 sets of observations were reportedly made. As can be seen in Figure 2, the correction to be applied varies widely depending most importantly on the age of the Moon (waxing or waning).

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[14] Cook, James (1769). *Journal of the Voyage of the Endeavour*. Admiralty MS in the Public Record Office, London. Adm 55/40. Cited in Beaglehole, J.C., ed. (1955). Cambridge University Press, p.73.

[15] Bayly, William. (1782). *The Original Astronomical Observations made in the course of a Voyage to the Northern Pacific Ocean.* Commissioners of Longitude, London. pp.27–28. Bayly gives a mean value of 210° 29'8½"E (149°30.9'W) but the correctly calculated mean is 210°30'11"E (149°29.8'W).

[16] Green, Charles, & Cook, James. (1771). *Observations made ... at King George's Island ... by Mr. Charles Green ... and Lieut. James Cook .... Philosophical Transactions of the Royal Society* 61, pp.407–408. The given mean of 149°36'39"W is the mean of each day's observations. The mean of the sets, which is how Vancouver would have calculated it, is 149°34'56"W (149°34.9'W). William Wales (1777) accepted 149°34'49½"W (149° 34.8'W) as being correct. Green died during the voyage home.

[17] Doe, N.A. (1993). *An Analysis of Captain Cook's Longitude Determinations at Nootka, April 1778*. Journal of the Canadian Hydrographic Association. *Lighthouse* no.48. pp.21–29.

[18] Standish E.M., et al. (1992). *Orbital Ephemerides of the Sun, Moon, and Planets*. In *Explanatory Supplement to the Astronomical Almanac*. Siedelmann P.K., ed. University Science Books, California. pp.70–84.

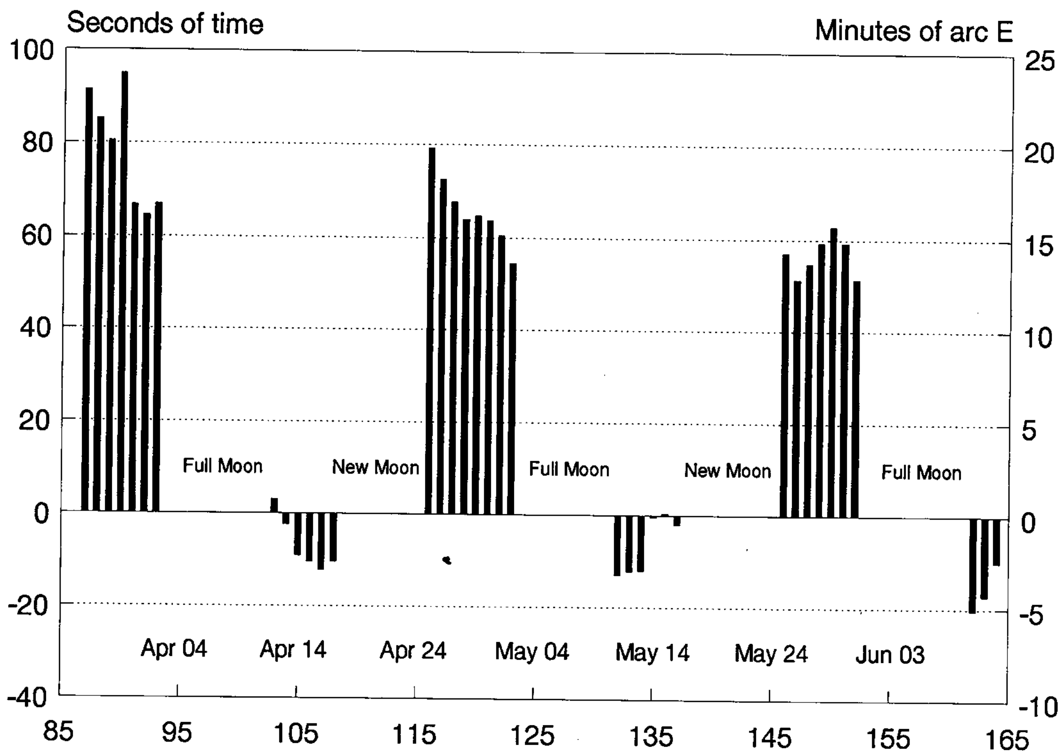

FIGURE 2 Nautical Almanac (1792) lunar-distance timing corrections. A positive correction signifies actual time is later than Almanac time. The equivalent terrestrial-longitude error is shown on the scale on the right.

Lunar distance was computed from theoretical determinations of the ecliptic longitude of the Sun, and the ecliptic longitude and latitude of the Moon. Examination of the values of the ecliptic longitudes and latitudes given in the Nautical Almanac shows that whilst there were errors in all three quantities involved, the error in the longitude of the Moon was the most significant error. This is shown in Figure 3 for March 29 (RJD:+80) to May 4 (RJD:+125).

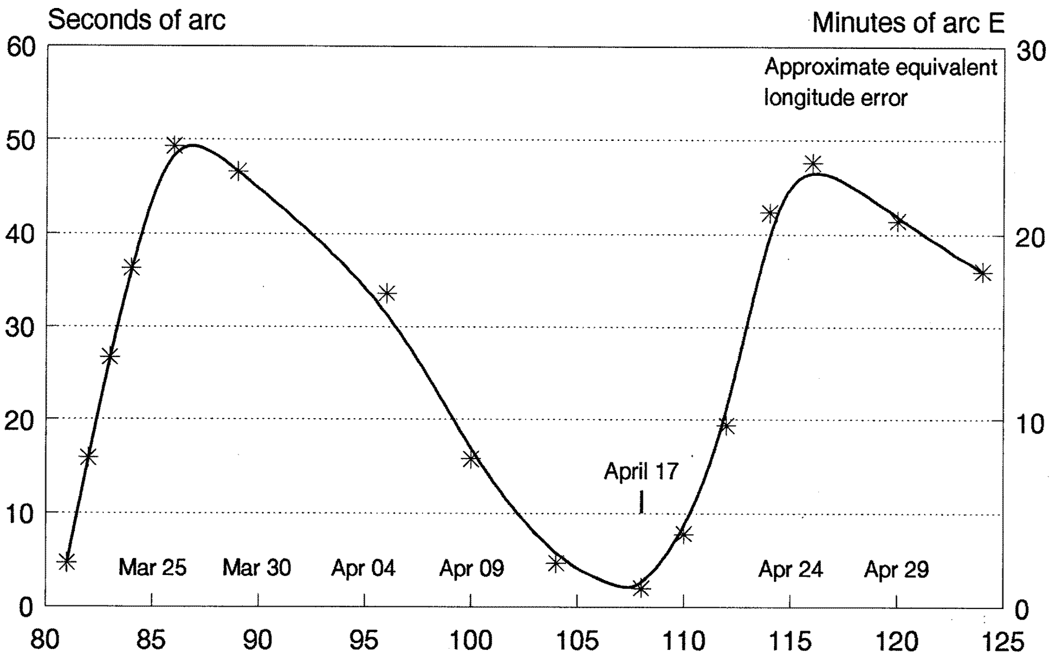

FIGURE 3 Nautical Almanac (1792) Moon ecliptic-longitude error determined using a modern ephemeris computed by the JPL. The equivalent terrestrial-longitude error is shown on the scale on the right.

The period of the oscillation shown in Figure 3 is about 29 days, i.e. a synodical month (new-moon to new-moon). The peak-to-peak error is about 47" and the positive off-set is almost exactly half of this at 25". This off-set, neglecting the small errors in solar longitude and lunar latitude, is approximately equivalent to an off-set in longitude determinations of 12.5'. Moreover, since the error is positive, an observation of the Moon's position would appear to have been made at an earlier time than it actually was, and the observer would consequently set his Greenwich chronometer slow, and all his longitude determinations would be too far east. This, I am sure, is the origin of Vancouver's error.

RJD	Vancouver's Date 1792	Observer	No. of sets	Vancouver's Longitude	Correction	Corrected Longitude
119	Apr. 28	Puget	9	122°40.9'W	+16.0'	122°56.9'W
86 - 164	Mar.26 - Jun.12	Whidbey	58	122°36.4'W	+ 7.9'	122°44.3'W
86 - 164	Mar.26 - Jun.12	Orchard	53	122°38.0'W	+ 7.5'	122°45.5'W
87 - 120	Mar.27 - Apr.29	Stewart	24	122°34.2'W	+14.2'	122°48.4'W
87 - 120	Mar.27 - Apr.29	Ballard	38	122°37.8'W	+14.0'	122°51.8'W
88 - 126	Mar.28 - May 5	Vancouver	38	122°38.9'W	+13.4'	122°52.3'W
			220			
	collective mean =			122°37.4'W	+10.8'	122°48.2'W
					modern value is	122°52.5'W
Notes:	Vancouver's calculation of the collective mean was almost exactly right; he gives 122°37.7'W. Corrections assume lunar distances were measured from the Sun. Puget's data is contained in his rough log and is not correctly dated by Vancouver. Corrections are weighted means for days on which measurements were made, or possibly made.					

TABLE 2 Vancouver's Port Discovery longitude corrected

Unfortunately it is impossible to exactly correct Vancouver's Port Discovery longitude without knowing the precise date on which each of the 220 sets of observations was made. Nevertheless, in Table 2 I have attempted a correction by averaging correction factors over the days when the observations might have been made. The table shows a new collective mean of 122°48.2'W, which is about 11' west of Vancouver's determination, and 4' east of the correct position at 122°52.5'W. The uncertainty in the corrections because of the fluctuations in the Nautical Almanac error over the course of a month, and the uncertain dates, is –7.9' to +6.8'; however what for the author is especially pleasing is the result for Vancouver's observations. Except for Peter Puget's,[19] these can be more closely dated than those of his officers. Vancouver himself, according to my reckoning, given an accurate Almanac, would have put the Port Discovery anchorage at 122°52.3'W. This is no more than a few hundred metres from its true position.

The extent to which Vancouver's error at Port Discovery influenced his subsequent longitude determinations is readily deducible from the Lamb edition of his book. The average error of twelve locations between Birch Bay and Menzies Bay is 16'E, i.e. the whole coastline was shifted eastwards.

13. Contemporary observations at Greenwich. Although Captain Vancouver was clearly a busy man on his return England, one wonders whether it would have been possible in theory for him to have resolved the difference between his chronometer and his lunar-distance longitude determinations by re-working his calculations using the observed positions of the Moon rather than the predicted positions in the Nautical Almanac. It turns out that in principle he could have. Following a procedure given in an appendix to an earlier paper,[6] I have compared the tabulated values of the Moon’s longitude in the Nautical Almanac with

[19] Doe, N.A. (1994). *Vancouver's strange reporting of longitude determinations by Peter Puget* (unpublished paper).

those calculated from observations made at the Greenwich Observatory in 1792. The source of the Greenwich data is a study made for the Admiralty published in 1848.[20]

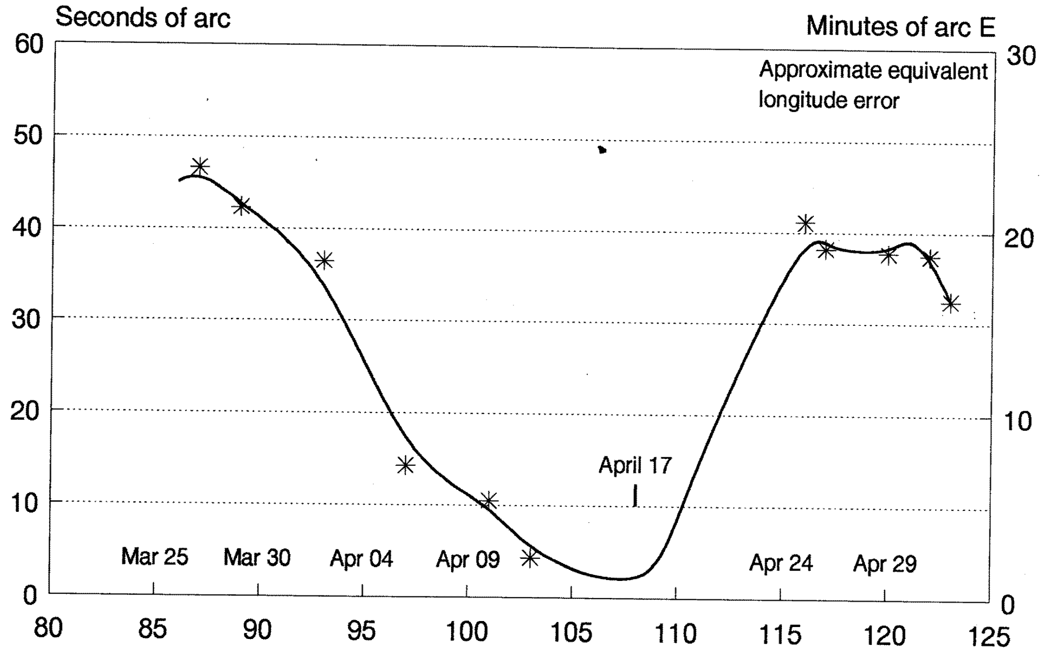

FIGURE 4 Nautical Almanac (1792) Moon ecliptic-longitude error determined from contemporary Royal Greenwich Observatory observations.

The results of the computation of the Moon's observed longitude are shown in Figure 4. For all practical purposes these are the same as those generated by the modern ephemeris and shown in Figure 3. It is unfortunate that there is a gap in the Greenwich records between April 12 and April 25, presumably because of bad weather; however, clearly, late 18th-century astronomers, given the inclination, could have arrived at the same resolution of Vancouver's problems that we have, 200 years later. Had they done so, it would certainly have been no surprise to Alexander von Humboldt, who correctly surmised in the early 1800s that the early lunar tables were inaccurate.[21] The Spanish naval officer Dionisio Alcalá Galiano would have been even less surprised, for it was he who reportedly discussed the existence of the errors in the Nautical Almanac with Vancouver at Nootka in the fall of 1792.[22]

14. Conclusion. This analysis of Vancouver's longitudes shows him to have been a master of his techniques, yet, tantalizingly, it gives no clue as to whether Vancouver fully understood the short-comings of the lunar-distance method. If he did, did he fail to mention

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[20] Lord Commissioners of the Admiralty. (1848). *Reduction of Observations of the Moon made at Greenwich from 1750 to 1830.* RGO Archives, Cambridge University.

[21] Humboldt, Alexander von. (1811). *Political Essay on the Kingdom of New Spain.* Black, J., trans. London. p. lvi.

[22] Museo Naval Madrid MS 619. (1793). *Voyage of Sutil and Mexicana 1792.* Kendrick, J., trans. (1991). Arthur H. Clark, Spokane, Washington. p. 215.

his concerns on his return to England for fear of antagonizing an already unfriendly Establishment, or did he just not “want to know” because of the immense amount of work he and Cook had invested in their longitude calculations? Had he lived longer we might have known, but as it is we can only hope that a closer look at the remainder of Vancouver's survey data will help to provide an answer.

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