Wayfinding in the woods using shadows

[Note: Figures are for Gabriola Island, BC in 2019 at the arbitrarily chosen location 49º08’N, 123º46.5’W chosen only because the computer needed to know when and where. The difference for other years and other places on the island are very small.]

If you go into the woods on Gabriola and venture off the main trails, you need to have a modicum of navigational skills to avoid getting lost. On sunny days with no clouds in the forecast and no possibility you’ll be out after dark, you can easily get by using the sun and a wrist watch. Actually looking at the sun directly is a decidedly bad idea, it does your eyes no good, but your shadow is right there and does just as well.

The very simplest rule-of-thumb about a shadow is that it points north (0º) at noon, and rotates clockwise at an average of 15º per hour. A practical way of estimating 15º is the one used by astronomers; the angle subtended by your clenched fist with tucked-in thumb at arm’s length.

If you check azimuths reckoned this way against your compass however, you might be surprised at the difference. It’s not that your compass that’s wrong, assuming you’ve remembered that magnetic north is 16º east of geographic north, it’s the irregularities of the annual and semi-annual movements of the sun.

In practice, <u>you do not need to worry about any of this</u>, your track through the woods is likely to zig and zag anyway as you go around deadfalls, bypass thickets of salal and roses, steer clear of thistles and stinging nettles, and take the advice of the deer. It’s usually only the general direction you’re moving that counts because you’re aiming for a trail, road, or any other familiar linear geographic feature. However, just out of interest, here are the reasons why your shadow might be misleading.

Daylight saving time

PDT (Pacific Daylight Time) is not something the sun knows anything about. In the summer, “noon” by your watch will be 1 o’clock, not twelve. You have to subtract an hour from PDT to get PST (Pacific Standard Time), which is designed to be close to mean solar time.

Time zone and longitude

Your watch will be set for PST in winter, or PDT in summer, which is eight hours and seven hours behind UTC respectively. But this is only correct for the mean position of the sun at longitude 120ºW. On Gabriola, your watch will be 15 minutes ahead of apparent solar time because we’re about 300 kilometres west of the PST meridian, which is just east of Kamloops.

24-hour clocks

Before getting into the nitty-gritty of why shadows are not perfect clocks beyond PDT and longitude, let’s review why we take the duration of a mean solar day to be 24 hours.

The earth rotates on average counter-clockwise around its axis (north at the top) once every 23 hours and 56 minutes. Although this time is not perfectly constant, we can safely assume for present purposes that it is.

The additional four minutes we add to this “stellar day” (the day relative to the stars) is to allow the position of the sun to catch up with the position it had the previous day, which is at noon, due south, and the highest it gets that day. During the stellar day, the sun appears in the geocentric world we live in to rotate also counter-clockwise making a complete circuit once a year. So having completed one rotation around its own axis, the earth needs to spin for a little bit longer before it faces the sun as it did at noon the previous day.

This additional four minutes is not constant for reasons explained below. If the sun is moving at a rate slightly above average, noon will appear early by our watch; we won't have caught up yet and the watch will seem to be too fast. Conversely, when the sun is moving at a rate slightly below average, noon will appear late by our watch; by the time the four minutes is up, we'll have moved passed noon, the watch will seem to be too slow.

Equation of time

The equation of time gives the relationship between mean solar time and time measured by the position of the sun. The equation has two components. Their combined effect is to put watch time out by no more than 14 minutes and then only in February. In summer, the difference between solar and watch time is never more than seven minutes, so you can safely ignore all of the following if it is of no interest. Nobody I know carries the equation of time in their head to avoid getting lost in the woods!

The following is only my personal explanation of the equation of time; me making sure I understand it. If you're interested in a more academic and precise explanation, I'd advise you to go to WIKIPEDIA instead. WIKIPEDIA and other easy-to-find web sites have some nice graphics that I won't repeat here.

Effect of Orbit Eccentricity

One of the two components of the equation of time is due to the fact that the earth's annual orbit around the sun is not perfectly circular, it's slightly elliptical. The effect however is very small; it's not the eccentricity of the earth's orbit that determines our annual seasons. If it did, mid-summer in the northern hemisphere would currently be close to New Year's Day.

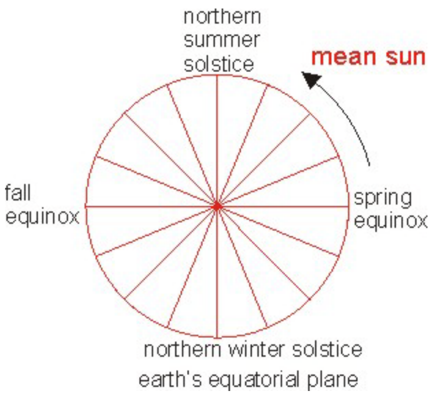
When the distance between earth and the sun is at its lowest (perihelion), currently in the northern winter, the earth, and hence the apparent position of the sun, is moving faster than at other times of the year as the earth swings by the sun. Conversely, when the distance between earth and the sun is at its highest, currently in the northern summer, the earth, and hence the apparent position of the sun, is moving slower than at other times of the year.

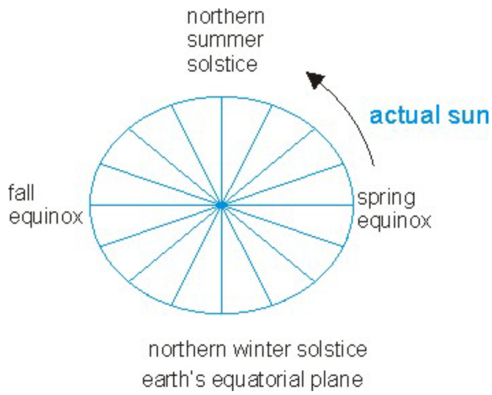
The cumulative effect of this is to make your watch fast by a maximum of about seven minutes at the beginning of April, and slow by the same amount at the beginning of October. Around mid-summer and around mid-winter, the effect is only a few minutes or less either way and is negligible for present purposes.

Effect of Obliquity

The second component of the equation of time is due to the fact that the plane of the earth's annual orbit around the sun (the ecliptic) is tilted a little less than 23.5º with respect to the equatorial plane of the earth, which is our reference plane for measuring azimuth. It is this tilt that primarily gives us our seasons.

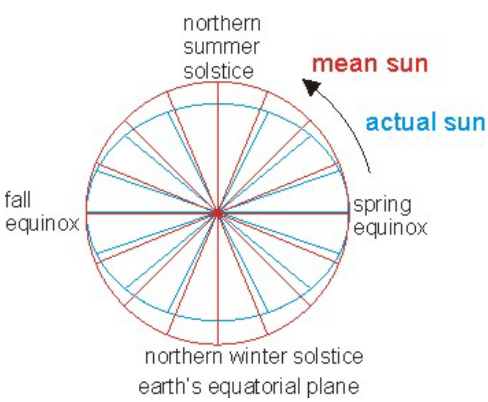
There are lots of web sites that show a graph of the fairly simple algebraic equation that deals with the effect of obliquity, but I found it not so easy to work out the geometry in my head, so here's how I think about it. It's a simplified explanation, not meant to be taken too seriously, but it seems to work.





The figure on the left (red) shows the movement during the year of the fictitious mean sun. At the centre is the earth. The plane of the mean sun's orbit is the same as the equatorial plane of the earth (there are no seasons), and the sun moves around the celestial equator counter-clockwise in a perfect circle and at a constant speed.

The figure on the right (blue) shows the same movement during the year of the factious mean sun except that the plane of the mean sun's orbit is now tilted with respect to the earth's equatorial plane and what is shown is the projection of this titled plane on to the earth's equatorial plane. The axis of the tilt is the horizontal line joining the equinoxes. The projected movement now appears elliptical, just does any circular object when viewed at an angle.



Putting these two diagrams together, as shown on the left, shows that, because of the tilt, time according to the mean sun (red, left diagram) occurs later than time due to the observed sun (blue, right diagram) when the sun is moving from the spring equinox in March to the summer solstice in June, and the time according to the mean sun (red, left diagram) occurs earlier than time due to the observed sun (blue, right diagram) when the sun is moving from the summer solstice to the fall equinox in September. This semi-annual cycle then repeats itself from the fall equinox back to the spring equinox of the following year.

A watch, which tracks mean solar time, is thus slow in early May and again six months later in early November, and is fast in early August and again six months later in early February. The magnitude of the peak difference amounts to almost ten minutes in all cases.

Combined effect

The combination of the Effects of Orbit Eccentricity and of Obliquity is the Equation of Time.

Watch Time at Noon

The combination of all of the above gives the watch time at noon when your shadow points directly north on Gabriola as follows. For each month the times are on the first and fifteenth. Daylight saving time (PDT) is taken as being between April 1 and October 15 (unshaded), which is not exactly right but you can figure that out. The accuracy is about plus or minus a minute:

1st & 15th		watch at noon
Jan.	PST	12:18
	PST	12:24
Feb.	PST	12:28
	PST	12:29
Mar.	PST	12:28
	PST	12:24

1st & 15th		watch at noon
Apr.	PDT	1:19
	PDT	1:15
May	PDT	1:12
	PDT	1:11
Jun.	PDT	1:13
	PDT	1:16

1st & 15th		watch at noon
Jul.	PDT	1:19
	PDT	1:21
Aug.	PDT	1:22
	PDT	1:20
Sep.	PDT	1:15
	PDT	1:10

1st & 15th		watch at noon
Oct.	PDT	1:05
	PDT	1:00
Nov.	PST	11:59
	PST	12:00
Dec.	PST	12:05
	PST	12:11

Rate of change of azimuth

At this point in the story many accounts will assume you have all or more than you wanted to know, but there is an important piece missing. This concerns the 15º per hour rule-of-thumb for times that are an hour or two before or after noon. This rule at our latitude is seriously wrong during the summer. The reason for this is that the plane of the earth's annual orbit around the sun (the ecliptic) is tilted with respect to the plane of our horizon, which is our reference for measuring azimuth. You can get a feel of why this matters if you consider how the sun moves at the earth's equator and at the earth's poles at an equinox in mid-March or mid-September.

At the equator on those days, the sun rises in the east, passes directly overhead at noon, and sets in the west. Its azimuth thus flips from east to west in an instant with nothing in between. However, at the poles on those days, the sun circles the horizon at a steady rate just like a clock.

At our latitude, the rule-of-thumb of 15º per hour is actually about 11º per hour of azimuth at sunrise and sunset throughout the year. But for us, the equator situation is approached in mid-summer as the sun is then nearest to being directly overhead at noon. It moves at about 32º per hour of azimuth at (solar) mid-day in mid-summer.

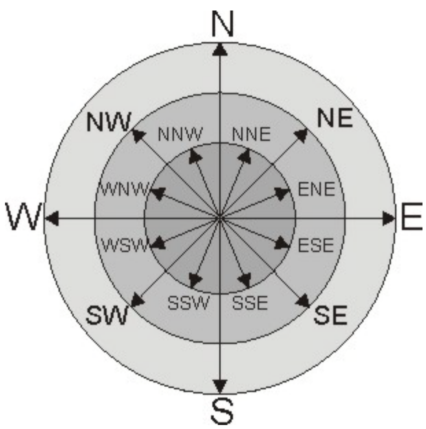
The polar situation is approached in mid-winter as the sun is then lowest in the sky at noon. For us, the rule-of-thumb of 15º per hour is good; the sun moves at about 14º per hour of azimuth at mid-day.

It is undoubtedly the summer figure that is most surprising. If you were to reckon, say, 11 o'clock by your watch as being "close enough" to noon, ignoring PDT, your shadow would still be west of northwest (WNW, 304º, W34ºN), almost 60º away from north.

Representing azimuth

In the tables below, there are three ways of representing azimuth.

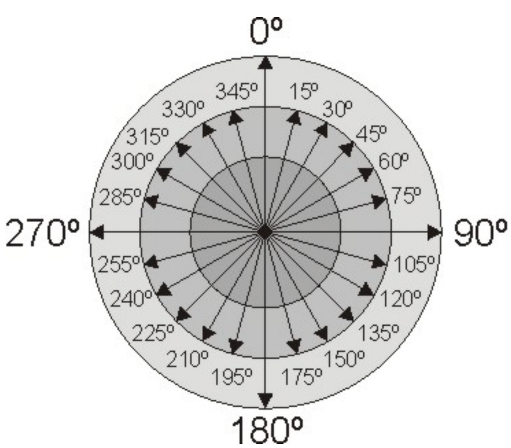
Compass rose



The four cardinal points of the compass rose (N, E, S, W) are in everyday usage. Also fairly common is the use of the additional four intercardinal directions (NE, SE, SW, NW). But from then on azimuths are rarely expressed by other than old-fashioned mariners this way. I've shown the 16-point rose, which very likely offers too many options for present purposes.

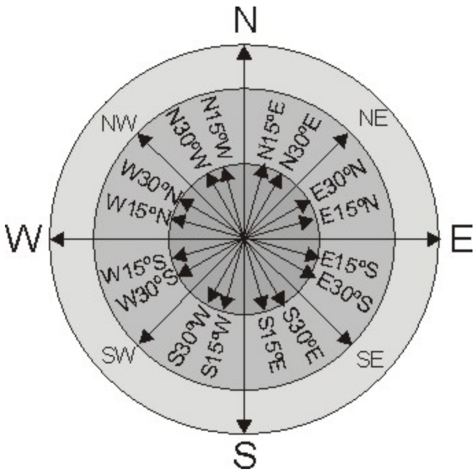
The problem with the compass rose system is that describing an azimuth with high precision is arithmetically awkward; who knows for example that "northeast by east" (NEbE) in the 32-point rose is logically NENE or numerically 56.25º.

Angular direction



The 0º to 360º system measured clockwise from north offers definition of direction with unlimited precision and is easily understood by computers, but it suffers for practical purposes in asymmetry. A bearing of 50º for example may be easily recognizable as being a little east of northeast, but its mirror image, 310º, a little to the west of northwest, is not so quickly recognized by most without doing the mental arithmetic 360º–310º = 50º.

Hybrid system



A hybrid system that I myself like to use in articles like this uses the four cardinal points (N, E, S, W) as reference points from which angular directions are measured both clockwise and counter-clockwise.

If exactness dictates that the intercardinal points (NE, SE, SW, NW) also be used, they can be. Thus for example N45ºE, which is the same as E45ºN, may simply be written NE.

Tables

There are three corresponding to the different ways of measuring azimuth. As before, for each month the times are on the first and fifteenth. Daylight saving time (PDT) is taken as being between April 1 and October 15 (unshaded), which is not exactly right but you can figure that out. The table for rates of change of azimuth only gives values for the first of each month.

Rules of thumb

I cannot imagine anyone wanting to carry a copy of one these tables when they go bush-whacking, and for all I know, if they did, there's probably an "app." to do that. My own rules of thumb are that taking "noon" to be one o'clock by the watch regardless of time of year is not a bad guide provided it is remembered that 15º per hour is only good in winter when PST is in effect. In summer, when PDT is in effect, a better rough estimate for middle of the day is 30º per hour.

A better rule than any of these is "always carry a compass". Small cheap ones designed for key-rings are quite adequate and you don't need tables to use it. Just remember though, actually putting it on a key-ring is not so smart if any of your keys or the ring itself is magnetic.

2019		AM 8	AM 10	NOON 12	PM 1	PM 2	PM 4	PM 6	PM 8
Jan.	PST		NNW	N	N	NNE	NE		
	PST		NW	N	N	NNE	NE		
Feb.	PST	WNW	NW	N	N	NNE	NE		
	PST	WNW	NW	N	N	NNE	NE		
Mar.	PST	WNW	NW	N	N	NNE	NE		
	PST	WNW	NW	N	N	NNE	ENE	E	
Apr.	PDT	W	WNW	NNW	N	NNE	NE	ENE	
	PDT	W	WNW	NNW	N	NNE	NE	E	ESE
May	PDT	W	WNW	NNW	N	NNE	ENE	E	ESE
	PDT	W	WNW	NNW	N	NNE	ENE	E	ESE
Jun.	PDT	W	WNW	NW	N	NNE	ENE	E	ESE
	PDT	W	WNW	NW	N	NNE	ENE	E	ESE
Jul.	PDT	W	WNW	NW	N	NNE	ENE	E	ESE
	PDT	W	WNW	NW	N	NNE	ENE	E	ESE
Aug.	PDT	W	WNW	NW	N	NNE	ENE	E	ESE
	PDT	W	WNW	NNW	N	NNE	ENE	E	ESE
Sep.	PDT	W	WNW	NNW	N	NNE	NE	E	
	PDT	W	WNW	NNW	N	NNE	NE	ENE	
Oct.	PDT	WNW	NW	NNW	N	NNE	NE	ENE	
	PDT	WNW	NW	NNW	N	NNE	NE	ENE	
Nov.	PST	WNW	NNW	N	NNE	NNE	ENE		
	PST	NW	NNW	N	NNE	NNE	NE		
Dec.	PST	NW	NNW	N	NNE	NNE	NE		
	PST		NNW	N	NNE	NNE	NE		

2019		AM 8	AM 10	NOON 12	PM 1	PM 2	PM 4	PM 6	PM 8
Jan.	PST		328	355	10	24	49		
	PST		326	354	9	23	49		
Feb.	PST	298	323	353	8	23	51		
	PST	295	321	352	9	25	53		
Mar.	PST	293	319	352	10	27	56		
	PST	290	317	352	11	30	60	84	
Apr.	PDT	275	300	333	353	14	51	78	
	PDT	272	297	332	354	17	55	82	105
May	PDT	269	294	330	355	20	60	87	109
	PDT	266	291	328	355	22	63	90	111
Jun.	PDT	264	288	325	353	23	66	92	113
	PDT	262	286	323	352	23	66	92	114
Jul.	PDT	262	285	322	350	21	65	91	113
	PDT	262	286	323	350	19	63	90	111
Aug.	PDT	265	289	325	350	17	60	87	109
	PDT	268	293	328	352	17	57	84	107
Sep.	PDT	273	298	333	354	17	54	81	
	PDT	277	303	336	356	17	52	78	
Oct.	PDT	283	308	340	359	17	50	75	
	PDT	287	312	342	0	17	48	73	
Nov.	PST	303	329	0	16	32	58		
	PST	305	330	0	15	30	55		
Dec.	PST	306	330	359	14	28	53		
	PST		330	358	12	26	51		

2019		AM 8	AM 10	NOON 12	PM 1	PM 2	PM 4	PM 6	PM 8
Jan.	PST		N32°W	N5°W	N10°E	N24°E	E41°N		
	PST		N34°W	N6°W	N9°E	N23°E	E41°N		
Feb.	PST	W28°N	N37°W	N7°W	N8°E	N23°E	E39°N		
	PST	W25°N	N39°W	N8°W	N9°E	N25°E	E37°N		
Mar.	PST	W23°N	N41°W	N8°W	N10°E	N27°E	E34°N		
	PST	W20°N	N43°W	N8°W	N11°E	N30°E	E30°N	E6°N	
Apr.	PDT	W5°N	W30°N	N27°W	N7°W	N14°E	E39°N	E12°N	
	PDT	W2°N	W27°N	N28°W	N6°W	N17°E	E35°N	E8°N	E15°S
May	PDT	W1°S	W26°N	N30°W	N5°W	N20°E	E30°N	E3°N	E19°S
	PDT	W4°S	W21°N	N32°W	N5°W	N22°E	E27°N	E	E21°S
Jun.	PDT	W6°S	W18°N	N35°W	N7°W	N23°E	E24°N	E2°S	E23°S
	PDT	W8°S	W16°N	N37°W	N8°W	N23°E	E24°N	E2°S	E24°S
Jul.	PDT	W8°S	W15°N	N38°W	N10°W	N21°E	E25°N	E1°S	E23°S
	PDT	W8°S	W14°N	N37°W	N10°W	N19°E	E27°N	E	E21°S
Aug.	PDT	W5°S	W19°N	N35°W	N10°W	N17°E	E30°N	E3°N	E19°S
	PDT	W2°S	W23°N	N32°W	N8°W	N17°E	E33°N	E6°N	E17°S
Sep.	PDT	W3°N	W28°N	N27°W	N6°W	N17°E	E36°N	E9°N	
	PDT	W7°N	W33°N	N24°W	N4°W	N17°E	E38°N	E12°N	
Oct.	PDT	W13°N	W38°N	N20°W	N1°W	N17°E	E40°N	E15°N	
	PDT	W17°N	W42°N	N18°W	N	N17°E	E42°N	E17°N	
Nov.	PST	W33°N	N31°W	N	N16°E	N32°E	E32°N		
	PST	W35°N	N30°W	N	N15°E	N30°E	E35°N		
Dec.	PST	W36°N	N30°W	N1°W	N14°E	N28°E	E37°N		
	PST		N30°W	N2°W	N12°E	N26°E	E39°N		

2019		AM 8	AM 10	NOON 12	PM 1	PM 2	PM 4	PM 6	PM 8
Jan.	PST		13	14	14	14	12		
Feb.	PST	11	14	16	16	15	12		
Mar.	PST	12	15	18	18	16	13		
Apr.	PDT	11	14	19	21	21	16	12	
May	PDT	11	14	23	26	24	16	12	11
Jun.	PDT	11	14	25	30	28	16	11	11
Jul.	PDT	11	14	25	31	29	16	11	11
Aug.	PDT	11	14	23	27	26	16	12	11
Sep.	PDT	11	14	20	23	22	16	12	
Oct.	PDT	12	14	18	19	18	14	12	
Nov.	PST	12	15	16	16	15	12		
Dec.	PST	11	13	15	14	14	12		

degrees per hour