NOTE: The print version of this article had black-and-white illustrations. The main text and page numbering in this version is the same, but there may have been very minor alterations to the figure captions.

#### Context:

Salt-weathering, sandstone, Nanaimo Group, geometry of tafoni, cavernous weathering, alveolization, erosion patterns, Thiessen polygons, fretting, Malaspina Gallery.

#### Citation:

Doe, N.A., The geometry of honeycomb weathering of sandstone, *SHALE* 26, pp.31–60, November 2011.

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#### Errors and omissions:

Appendix 4 originally contained some serious arithmetic errors. A corrected version was posted on August 31, 2015.

Earlier references: http://www.nickdoe.ca/pdfs/Webp26c.pdf http://www.nickdoe.ca/pdfs/Webp51c.pdf http://www.nickdoe.ca/pdfs/Webp512c.pdf also http://www.nickdoe.ca/pdfs/Webp555c.pdf

Date posted: November 7, 2011.

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# The geometry of honeycomb weathering of sandstone

#### by Nick Doe

Sandstone honeycombing—otherwise known as *tafoni*, cavernous weathering, alveolization, and sometimes fretting, boxwork, and fluting—is often said to be the result of erosion by the wind and the waves. The undoubted culprit here in the Canadian Gulf Islands however, is salt.<sup>12</sup>

Sandstone is relatively coarse-grained and porous—you can't spit-and-polish it as you can igneous-rock pebbles on the beach—and it absorbs water like a paper towel. When sandstone is exposed to the sun, any saline water within the pores of the rock is drawn to the surface where salt crystallizes as the water evaporates. When salt crystallizes from a supersaturated solution in a confined space, it exerts pressure,<sup>3</sup> and this pressure is

<sup>3</sup> The explanation for "crystallization pressure" is anything but simple-just try Googling the phrase. Essentially though, I think it goes like this. When a molecule of salt comes out of an evaporating solution and joins a crystal it is because the attractive electrostatic-force exerted by the diminishing number of water molecules has become less than the attractive electrostatic-force exerted by the crystal. However, within a pore, the orientation and positioning of the arriving salt molecule is different from what it would be without the presence of the pore wall, which itself, on a small enough scale, carries electrostatic charges just like the crystal. This "less than optimal" orientation and positioning of the newly arrived molecule of salt expresses itself as being the result of a repulsive electrostatic-force between the crystal and the wall. This repulsive

enough to create new microfractures, rejuvenate old ones, and thereby weather the rock.

Honeycombing mostly occurs on inclined surfaces facing the sun, a fact that was brought home to me when we recently visited Australia and noted the nearcomplete absence of honeycombing facing south, the reverse of what happens here in the northern hemisphere (see Appendix 3).

A fundamental step in understanding the geometry of honeycombing is to realize that it is not determined by the pattern of surface wetting, rather it is dictated by movement of water *from within* the rock to the surface. Random wave and wind-borne splashes could not possibly create the well-ordered patterns that are commonly observed.

Honeycombing occurs only on porous rocks—you seldom see it on shale—but some normally impermeable rocks are sometimes honeycombed because they have become porous as the result of chemical weathering, or because they are volcanic and contain microvesicles.<sup>4</sup>

Sandstone in the Gulf Islands becomes salty as a result of being immersed at high tide; by wicking up seawater by capillary action; by being wetted by waves; by absorbing

force manifests itself as crystallization pressure that forces the crystal to assume the shape of the pore rather than the shape it would have if unrestricted. <sup>4</sup> I've seen examples in New Mexico in poorlywelded volcanic tuff (*rhyollite*) and in Australia in *granitic pegmatite* and I'm told it occurs in *basalt* [Dave Tucker & George Mustoe].

<sup>&</sup>lt;sup>1</sup> Doe N., *What makes holes in sandstone*, pp.12–40, *SHALE* 9, August 2004.

<sup>&</sup>lt;sup>2</sup> Doe N., *Salt-weathering of upper Nanaimo Group sandstone*, pp.34–56, *SHALE* 23, March 2010.

spray; and by "groundwater" containing salt making its way to cliff faces in seepages. Groundwater moving down near-vertical fractures a short distance behind the faces of cliffs often give rise to particularly dense and spectacular clusters of honeycomb holes and, in the extreme, to cavernous galleries big enough to walk under.<sup>5</sup>

The development of honeycombing in the intertidal zone in the Gulf Islands is helped by the especially favourable pattern of tides in the Strait of Georgia (Salish Sea). These have a strong solar diurnal component. Low tide in the summer is often when the sun is strongest during the day giving the rock ample opportunity to dry out before being wetted again during the high tide at night.

Although conditions within the honeycombs may not always favour evaporation—low air circulation and high humidity—no matter how low the evaporation rate is, it almost always exceeds the rate at which water diffuses through the rock to the surface (see Appendix 4).

A few years ago, I wrote a paper on honeycombing<sup>1</sup> and in support of my ideas as to what causes the holes, I argued that they distribute themselves over the surface of the sandstone in a hexagonal pattern. Mathematically, this is the best way of packing as many non-overlapping, equalsized circles on a plane as possible.

While there is nothing fundamentally wrong with this idea, it suffers from two awkward facts. One is that if you look at a small selection of holes, they seldom conform to anything like a near-perfect hexagonal array. The second is that it fails to explain how the holes co-operate among themselves to arrive at this geometrically-perfect arrangement.



Above: Development of honeycomb holes. Water within rock exposed to the sun moves from within the rock (*bottom right*) toward the surface driven by differential capillary pressure where the water evaporates and deposits salt. Depressions provide an easy route for the water to the surface and so more salt is deposited at the bottom of the holes than on surrounding surfaces. This deepens the holes.

*Below*: Some sandstone surfaces have been case-hardened, making them more resistant to weathering. The holes then develop as cavities that act like ovens. Some boulders are almost completely hollowed out this way.



<sup>&</sup>lt;sup>5</sup> Doe N., *The Malaspina Galleries*, pp.53–56, *SHALE* 9, August 2004. The Galiano Gallery on Gabriola is one of a row of cavernous holes, which is why locals persist in calling them "galleries" (plural).



*Left*: Really big honeycomb holes can become galleries especially if provided with a good supply of slightly salty groundwater flowing down near vertical fractures behind the cliff face.

*Below*: The Galiano Gallery, one of a set of galleries known as the Malaspina Galleries, is a merger of several such holes. In the photo below, the oft-visited big gallery is out of the picture on the left. This photo shows its accompanying row of caverns. The pitch-to-diameter ratio of the caverns is remarkably similar to that seen in far smaller distinctive rows of honeycomb holes, and there is a reason for this.





*Left*: Rows of holes often trace out bedding planes and fractures that are sources of moisture working its way to the surface. The bedding planes in the photo dip slightly to the right.

Hammer *upper centre* for scale.



*Opposite*: Honeycombing on the cliffs of Valdes Island show most of the stages of honeycomb evolution.

Salt water is supplied by "rockwater" containing traces of sea spray running down a near vertical bedding-plane-perpendicular fracture A (lower picture) a short distance back from the face. The fracture face is case-hardened.

**B** (lower picture) is part of the original surface that has also been case-hardened. Such features form the roofs of galleries.

**C** (lower picture) is one of several spherical concretions. These are cemented with *calcite* derived from fossil material rather than clay and consequently have a different porosity and permeability than the host rock. Concretions are also honeycombed, but they weather at a slower rate.

**D** (upper picture) is part of the face in the early stages of honeycombing. Small holes (pits) are scattered across the surface.

Gradually the holes get larger and deeper, E (upper picture). The holes increase their depth at first until evaporation from the bottom of the holes is diminished by lack of sunlight. Weathering of the walls of the holes however continues.

Eventually holes begin to merge F(upper picture) creating fretting. Rows of holes become slots. The thin walls of the holes are protected from erosion by being starved of salt by the holes either side of them. Almost no water from the interior of the rock diffuses up the walls to the surface.

When at their maximum depth, the walls of the larger holes are undermined **G** (upper picture).

The stripped surface H (upper picture) either starts a new cycle of honeycombing, or it erodes steadily at a uniform rate because the surface is, as far as evaporation is concerned, a uniform surface, or because there is so much salt present that variations in the concentration of salt across the surface no longer cause variations in erosion rates.

## The evolution of honeycomb patterns

Honeycombing represents an intermediate stage in the denudation of sandstone surfaces.<sup>6</sup> Weathering progresses from isolated pits, through the formation of geometrically arranged holes, the merging of holes to form larger ones, the amalgamation of rows and columns into slots and frets, and ultimately, the stripping of the honeycomb layer. Sometimes, the bottoms of large honeycomb holes develop smaller

honeycomb patterns and the cycle repeats itself as erosion proceeds deeper into the rock. At other times, the stripped layer erodes uniformly and no further patterning occurs. This is most noticeable on the back walls of large galleries. As noted in the photo caption, either evaporation and erosion is uniform across stripped surfaces, or the concentration of salt, which is usually clearly visible, is so high that variations in salt concentration no longer cause variations in erosion rates. The depth that a honeycomb hole reaches is limited by the reduction in efficiency of the floor of the hole as an "evaporation surface" as it becomes increasingly shaded from the sun by the walls of the hole.<sup>7</sup>

<sup>&</sup>lt;sup>6</sup> There are many published papers on the topic of honeycomb evolution, but the one I found most useful was: Mottershead D.N., *Spatial variations in intensity of alveolar weathering of a dated sandstone structure in a coastal environment, Weston-super-Mare, UK*, in Robinson D.A.& Williams R.B.G. (ed.), *Rock weathering and landform evolution*, pp.151–174, Wiley, 1994.

<sup>&</sup>lt;sup>7</sup> Some oval holes may have been shaped by variations in sunlight penetrating to the floor. See Appendix 1 and 2 for more detail.

### Isolated pits

The honeycombing process begins with isolated circular pits. An oftmade assumption is that it is the roughness of the rock surface that provides loci for the holes, and this maybe so in some cases, but because moisture is being drawn to the surface from inside the rock, I think equally, or more important, are internal asymmetries in permeability and porosity not necessarily visible at the surface. Sandstone surfaces that look "featureless and flat" aren't necessarily surfaces across which evaporation is everywhere the same.

In a sense, the pits are not unlike potholes in roads. You may repair a pothole by filling it and making the surface perfectly smooth, but it often happens that the pothole reappears in exactly the same place. Part of the reason the pothole is where it is has to do with a flaw in the subgrade of the road, not in the pavement.

Sandstone surfaces are seldom uniform in a hydrological sense, even when they are optically flat.

#### Arrays of holes

Once the density of holes becomes high enough, they begin to interact with each other. By acting as a vent for the internal moisture, the holes deprive the area immediately around them of evaporating moisture. By depriving their

perimeters of moisture, they also deprive their perimeters of damaging crystallizing salt, and so the holes deepen leaving their



*Top*: An array of honeycomb holes on a fairly smooth featureless surface.

*Below*: The pattern of holes can be closely matched by a set of circles of only two different sizes. Practically everywhere, the circles touch adjacent circles, but they don't overlap even though the circles have larger diameters than the holes. Each honeycomb hole clearly has a sharply defined "zone of influence" within which other holes do not intrude.

perimeters intact. This inert zone I call the "zone of influence".

Arrays of holes often form rows and columns for various reasons.





The photograph shows the ancient surface of a desert. The white areas are small stones. You can see that the stones stand proud of the surface because they protect the sand underneath from being eroded away. Furrows run from top to bottom in the picture, and there are hints that the stones in them have been aligned (imbricated) by very occasional heavier flows of water— Well no. Actually I lie — the photograph is really a black-and-white-reversed picture of honeycomb holes. The runnels are hummocks, and the stones are holes. The patterning emphasizes how the distribution and shape of the holes is not random and due to properties of the rock not readily apparent to the naked eye.

Because of the way fine non-spherical particles settle out of suspension, sandstone often exhibits anisotropic permeability, that is, it is more permeable parallel to the bedding plane than perpendicular to it.<sup>8</sup>

Successive strata of sandstone also frequently show differences in average grain size and in silt and clay content, which in turn affects their porosity and permeability. On Gabriola, sandstone bedding planes are usually horizontal ( $\pm 15^{\circ}$ ) and honeycomb holes in near-horizontal rows are common.

Another observed asymmetry results from fractures. Honeycomb holes frequently cluster around a fracture because it provides a reservoir of water to supply adjacent

<sup>&</sup>lt;sup>8</sup> Rudi Meyer, *Anisotropy of sandstone permeability*, CREWES Research Report—Vol. 14, Department of Geoscience, University of Calgary, 2002.





Left: unconsolidated sand in a cliff face in the estuary of the Snohomish River WA. During deposition, the bank of sediment, then below sea level, slumped. This has left a record in the sand of the ensuing turbulence. Above: curious circular patterns of holes in DeCourcy Fm. Nanaimo Gp. sandstone on Norway Island possibly reflect internal drainage patterns created in the late Cretaceous in a similar fashion (a Bouma sequence perhaps?). Note the clustering of holes around the vertical fracture.

evaporation surfaces. A fracture can also quickly drain an area of water so there are no honeycomb holes near the fracture, but I observe this less frequently.

On Gabriola, fractures in the sandstone are often oriented perpendicular to bedding planes, so near-vertical strings of honeycomb holes are not hard to find.

Several other features of the sandstone can lead to anisotropic permeability, including spherical calcareous concretions and remnants of "fluid escape pipes"—the upward climbing paths that water took when it was being squeezed out of the sand as it was being compacted during burial millions of years ago.





*Top*: Honeycomb holes in a circular arrangement on a featureless surface. These patterns are associated with spherical concretions, so it is a fair bet that subsurface concretions are playing a role here.

Above: Honeycombed concretions. It may seem that the honeycombing is more advanced in the concretions than in the host rock, but this is not so. The weathering of the host sandstone in the background has progressed beyond honeycombing to the point where its surface has been stripped.

*Left*: Fractures can encourage honeycombing (fracture *far left*) by harbouring a reservoir of salty moisture, or, less often, they can discourage it (*centre*) by draining the area, or by retaining too much water to evaporate between tides.

The two faces of both vertical fractures have been case-hardened which shows up as "thick-lips" at the surface.

## Models

The idea that a honeycomb hole has a "zone of influence" on the surface extending beyond its rim—to use an analogy familiar to Gabriolans who get their water from a well—is simple enough, but I wanted to construct a model so that I could derive some numerical data.

I used two models; one being a computer spreadsheet and the other a tray of coins. Unfortunately, the value of the coins was not sufficient for me to purchase some expensive software that would have done both jobs.



The central vertical slot—there's another half way to its left is a fracture whose surfaces have been case-hardened. In general though, the cell walls of honeycombs are protected by being deprived of salt, not by case-hardening.

The combination of vertical fractures and horizontal bedding planes helps produce square arrangements of holes. Look for walls forming a + rather than a Y. There's an example half way down the righthand edge next to the arrow.

#### The spreadsheet model

The purpose of the spreadsheet model was to help explore the relationships between the degree of depletion of moisture at the surface where evaporation takes place, the distance from the hole's centre, its depth, and its diameter.

To do this, I set up a matrix of spreadsheet cells representing a thin

vertical slice of rock through a honeycomb rock, as explained in the following diagrams.



Bedding planes are not always visible, and thick Gabriola sandstone commonly lacks such structure, but when present, bedding clearly influences honeycombing geometry. Variations in permeability and porosity from one stratum to the next result in different sizes of honeycomb holes. In some strata there aren't any holes.

1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	0.96	0.95	0.94	0.93	0.89	0.83	0.84			0.84	0.83	0.89	0.93	0.94	0.95	0.96	0.96
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	0.96	0.95	0.95	0.94	0.93	0.93	1.00	1.37	1.37	1.00	0.93	0.93	0.94	0.95	0.95	0.96	0.96
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	0.96	0.95	0.95	0.95	0.96	0.98	1.04	1.16	1.16	1.04	0.98	0.96	0.95	0.95	0.95	0.96	0.96
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	0.96	0.96	0.96	0.96	0.97	0.99	1.03	1.07	1.07	1.03	0.99	0.97	0.96	0.96	0.96	0.96	0.96
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	0.96	0.96	0.96	0.97	0.98	0.99	1.01	1.03	1.03	1.01	0.99	0.98	0.97	0.96	0.96	0.96	0.96
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	0.96	0.96	0.97	0.97	0.98	0.99	1.00	1.01	1.01	1.00	0.99	0.98	0.97	0.97	0.96	0.96	0.96
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	0.96	0.97	0.97	0.97	0.98	0.98	0.99	1.00	1.00	0.99	0.98	0.98	0.97	0.97	0.97	0.96	0.96
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.96	0.97	0.97	0.97	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.98	0.98	0.97	0.97	0.97	0.97	0.96
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.97	0.97	0.97
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.97	0.97	0.97
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.97	0.97	0.97	0.97
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98	0.98
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99	1.00
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.90	0.89	0.88	0.85	0.81	0.75	0.65	0.54			0.54	0.65	0.75	0.81	0.85	0.88	0.89	0.90	0.63	0.62	0.59	0.55	0.49	0.42	0.32	0.22			0.22	0.32	0.42	0.49	0.55	0.59	0.62	0.63
0.90	0.90	0.89	0.87	0.85	0.82	0.82	1.06			1.06	0.82	0.82	0.85	0.87	0.89	0.90	0.90	0.64	0.63	0.61	0.58	0.53	0.48	0.42	0.37			0.37	0.42	0.48	0.53	0.58	0.61	0.63	0.64
0.91	0.91	0.90	0.89	0.89	0.90	0.95	1.09	1.63	1.63	1.09	0.95	0.90	0.89	0.89	0.90	0.91	0.91	0.67	0.66	0.65	0.63	0.60	0.58	0.57	0.58			0.58	0.57	0.58	0.60	0.63	0.65	0.66	0.66
0.92	0.92	0.92	0.92	0.93	0.95	1.00	1.11	1.29	1.29	1.11	1.00	0.95	0.93	0.92	0.92	0.92	0.92	0.70	0.70	0.69	0.69	0.69	0.71	0.77	0.90			0.90	0.77	0.71	0.69	0.69	0.69	0.70	0.70
0.93	0.93	0.93	0.94	0.95	0.98	1.02	1.08	1.15	1.15	1.08	1.02	0.98	0.95	0.94	0.93	0.93	0.93	0.74	0.74	0.74	0.75	0.78	0.83	0.99	1.49			1.49	0.99	0.83	0.78	0.75	0.74	0.74	0.74
0.94	0.94	0.94	0.95	0.97	0.99	1.02	1.05	1.08	1.08	1.05	1.02	0.99	0.97	0.95	0.94	0.94	0.94	0.78	0.79	0.80	0.82	0.85	0.92	1.05	1.30	2.05	2.05	1.30	1.05	0.92	0.85	0.82	0.80	0.79	0.78
0.95	0.95	0.95	0.96	0.97	0.99	1.01	1.03	1.04	1.04	1.03	1.01	0.99	0.97	0.96	0.95	0.95	0.95	0.82	0.83	0.84	0.87	0.91	0.98	1.08	1.26	1.53	1.53	1.26	1.08	0.98	0.91	0.87	0.84	0.83	0.82
0.96	0.00		0.97	0.00	0.00	1.00	1.01	1.02	1.02	1.01	1.00	0.00	0.00	0.97	0.96	0.96	0.96	0.86	0.87	0.88	0.91	0.95	1.00	1.08	1.19	1.30	1.30	1.19	1.08	1.00	0.95	0.91	0.88	0.87	0.86
	0.90	0.96	0.37	0.30	0.33	1.00	1.01	1.02	1.02	1,01	1.00	0.99	0.98	0.37	0.50		0.50														0.07	0.00	0.91	0.90	0.89
0.96	0.96	0.96 0.97	0.97	0.98	0.99	0.99	1.01	1.02	1.02	1.01	0.99	0.99	0.98	0.97	0.97	0.96	0.96	0.89	0.90	0.91	0.93	0.97	1.01	1.07	1.13	1.18	1.18	1.13	1.07	1.01	0.9/	0.93	0.51		
0.96 0.97	0.96 0.97	0.96 0.97 0.97	0.97 0.97	0.98	0.99	0.99 0.99	1.00 1.00 1.00	1.02 1.01 1.00	1.02 1.01 1.00	1.01 1.00 1.00	0.99 0.99	0.99 0.99 0.98	0.98 0.98 0.98	0.97 0.97 0.97	0.97 0.97	0.96 0.97	0.96 0.97	0.89 0.92	0.90 0.92	0.91 0.94	0.93 0.95	0.97 0.98	1.01 1.01	1.07 1.05	1.13 1.09	1.18 1.11	1.18 1.11	1.13 1.09	1.07 1.05	1.01 1.01	0.97	0.93	0.94	0.92	0.92
0.96 0.97 0.97	0.96 0.97 0.97	0.96 0.97 0.97 0.97	0.97 0.97 0.98	0.98 0.98 0.98 0.98	0.99 0.99 0.98 0.98	0.99 0.99 0.99 0.99	1.00 1.00 1.00 0.99	1.02 1.01 1.00 0.99	1.02 1.01 1.00 0.99	1.01 1.00 1.00 0.99	0.99 0.99 0.99 0.99	0.99 0.99 0.98 0.98	0.98 0.98 0.98 0.98	0.97 0.97 0.98	0.97 0.97 0.97 0.97	0.96 0.97 0.97	0.96 0.97 0.97	0.89 0.92 0.94	0.90 0.92 0.94	0.91 0.94 0.95	0.93 0.95 0.97	0.97 0.98 0.99	1.01 1.01 1.01	1.07 1.05 1.04	1.13 1.09 1.06	1.18 1.11 1.07	1.18 1.11 1.07	1.13 1.09 1.06	1.07 1.05 1.04	1.01 1.01 1.01	0.97	0.93 0.95 0.97	0.94	0.92 0.94	0.92 0.94
0.96 0.97 0.97 0.97	0.96 0.96 0.97 0.97 0.97	0.96 0.97 0.97 0.97 0.97 0.98	0.97 0.97 0.98 0.98	0.98 0.98 0.98 0.98 0.98	0.99 0.99 0.98 0.98 0.98	1.00 0.99 0.99 0.99 0.99	1.00 1.00 0.99 0.99	1.01 1.00 0.99 0.99	1.01 1.00 0.99 0.99	1.01 1.00 1.00 0.99 0.99	1.00 0.99 0.99 0.99 0.99	0.99 0.99 0.98 0.98 0.98	0.98 0.98 0.98 0.98 0.98	0.97 0.97 0.98 0.98	0.97 0.97 0.97 0.97 0.98	0.96 0.97 0.97 0.97	0.96 0.97 0.97 0.97	0.89 0.92 0.94 0.95	0.90 0.92 0.94 0.96	0.91 0.94 0.95 0.97	0.93 0.95 0.97 0.98	0.97 0.98 0.99 0.99	1.01 1.01 1.01 1.01	1.07 1.05 1.04 1.02	1.13 1.09 1.06 1.04	1.18 1.11 1.07 1.05	1.18 1.11 1.07 1.05	1.13 1.09 1.06 1.04	1.07 1.05 1.04 1.02	1.01 1.01 1.01 1.01	0.97 0.98 0.99 0.99	0.93 0.95 0.97 0.98	0.94 0.95 0.97	0.92 0.94 0.96	0.92 0.94 0.95
0.96 0.97 0.97 0.97 0.98	0.96 0.97 0.97 0.97 0.97 0.98	0.96 0.97 0.97 0.97 0.98 0.98	0.97 0.97 0.98 0.98 0.98	0.98 0.98 0.98 0.98 0.98 0.98	0.99 0.99 0.98 0.98 0.98 0.98 0.99	0.99 0.99 0.99 0.99 0.99 0.99	1.00 1.00 0.99 0.99 0.99	1.01 1.00 0.99 0.99 0.99	1.01 1.00 0.99 0.99 0.99	1.01 1.00 1.00 0.99 0.99 0.99	1.00 0.99 0.99 0.99 0.99 0.99	0.99 0.99 0.98 0.98 0.98 0.98 0.99	0.98 0.98 0.98 0.98 0.98 0.98	0.97 0.97 0.98 0.98 0.98	0.97 0.97 0.97 0.98 0.98	0.96 0.97 0.97 0.97 0.98	0.96 0.97 0.97 0.97 0.97 0.98	0.89 0.92 0.94 0.95 0.97	0.90 0.92 0.94 0.96 0.97	0.91 0.94 0.95 0.97 0.98	0.93 0.95 0.97 0.98 0.98	0.97 0.98 0.99 0.99 0.99	1.01 1.01 1.01 1.01 1.01	1.07 1.05 1.04 1.02 1.02	1.13 1.09 1.06 1.04 1.03	1.18 1.11 1.07 1.05 1.03	1.18 1.11 1.07 1.05 1.03	1.13 1.09 1.06 1.04 1.03	1.07 1.05 1.04 1.02 1.02	1.01 1.01 1.01 1.01 1.01	0.97 0.98 0.99 0.99 0.99	0.93 0.95 0.97 0.98 0.98	0.94 0.95 0.97 0.98	0.92 0.94 0.96 0.97	0.92 0.94 0.95 0.97
0.96 0.97 0.97 0.97 0.98 0.98	0.96 0.97 0.97 0.97 0.97 0.98 0.98	0.96 0.97 0.97 0.97 0.98 0.98 0.98	0.97 0.97 0.98 0.98 0.98 0.98 0.99	0.98 0.98 0.98 0.98 0.98 0.98 0.98	0.99 0.98 0.98 0.98 0.98 0.99 0.99	0.99 0.99 0.99 0.99 0.99 0.99 0.99	1.00 1.00 0.99 0.99 0.99 0.99	1.01 1.00 0.99 0.99 0.99 0.99	1.02 1.01 1.00 0.99 0.99 0.99 0.99	1.00 1.00 0.99 0.99 0.99 0.99	1.00 0.99 0.99 0.99 0.99 0.99 0.99 0.99	0.99 0.98 0.98 0.98 0.98 0.98 0.99 0.99	0.98 0.98 0.98 0.98 0.98 0.98 0.98 0.99	0.97 0.97 0.98 0.98 0.98 0.98 0.99	0.97 0.97 0.97 0.98 0.98 0.98	0.96 0.97 0.97 0.97 0.98 0.98	0.96 0.97 0.97 0.97 0.98 0.98	0.89 0.92 0.94 0.95 0.97 0.98	0.90 0.92 0.94 0.96 0.97 0.98	0.91 0.94 0.95 0.97 0.98 0.98	0.93 0.95 0.97 0.98 0.98 0.99	0.97 0.98 0.99 0.99 0.99 1.00	1.01 1.01 1.01 1.01 1.01 1.00	1.07 1.05 1.04 1.02 1.02 1.01	1.13 1.09 1.06 1.04 1.03 1.02	1.18 1.11 1.07 1.05 1.03 1.02	1.18 1.11 1.07 1.05 1.03 1.02	1.13 1.09 1.06 1.04 1.03 1.02	1.07 1.05 1.04 1.02 1.02 1.01	1.01 1.01 1.01 1.01 1.01 1.00	0.97 0.98 0.99 0.99 0.99 1.00	0.93 0.95 0.97 0.98 0.98 0.98	0.94 0.95 0.97 0.98 0.98	0.92 0.94 0.96 0.97 0.98	0.92 0.94 0.95 0.97 0.98
0.96 0.97 0.97 0.97 0.98 0.98 0.99	0.96 0.97 0.97 0.97 0.98 0.98 0.99	0.96 0.97 0.97 0.98 0.98 0.98 0.98	0.97 0.97 0.98 0.98 0.98 0.98 0.99 0.99	0.98 0.98 0.98 0.98 0.98 0.98 0.99 0.99	0.99 0.98 0.98 0.98 0.98 0.99 0.99 0.99	1.00 0.99 0.99 0.99 0.99 0.99 0.99 0.99	1.00 1.00 0.99 0.99 0.99 0.99 0.99	1.01 1.00 0.99 0.99 0.99 0.99 0.99	1.02 1.01 1.00 0.99 0.99 0.99 0.99 0.99	1.00 1.00 0.99 0.99 0.99 0.99 0.99	1.00 0.99 0.99 0.99 0.99 0.99 0.99 0.99	0.99 0.99 0.98 0.98 0.98 0.99 0.99 0.99	0.98 0.98 0.98 0.98 0.98 0.98 0.99 0.99	0.97 0.97 0.98 0.98 0.98 0.98 0.99 0.99	0.97 0.97 0.97 0.98 0.98 0.98 0.98 0.99	0.96 0.97 0.97 0.97 0.98 0.98 0.98	0.96 0.97 0.97 0.97 0.98 0.98 0.99	0.89 0.92 0.94 0.95 0.97 0.98 0.99	0.90 0.92 0.94 0.96 0.97 0.98 0.99	0.91 0.94 0.95 0.97 0.98 0.98 0.99	0.93 0.95 0.97 0.98 0.98 0.99 0.99	0.97 0.98 0.99 0.99 0.99 1.00 1.00	1.01 1.01 1.01 1.01 1.01 1.00 1.00	1.07 1.05 1.04 1.02 1.02 1.01 1.01	1.13 1.09 1.06 1.04 1.03 1.02 1.01	1.18 1.11 1.07 1.05 1.03 1.02 1.01	1.18 1.11 1.07 1.05 1.03 1.02 1.01	1.13 1.09 1.06 1.04 1.03 1.02 1.01	1.07 1.05 1.04 1.02 1.02 1.01 1.01	1.01 1.01 1.01 1.01 1.01 1.00 1.00	0.97 0.98 0.99 0.99 0.99 1.00 1.00	0.93 0.95 0.97 0.98 0.98 0.99 0.99	0.94 0.95 0.97 0.98 0.98 0.98 0.99	0.92 0.94 0.96 0.97 0.98 0.99	0.92 0.94 0.95 0.97 0.98 0.99
0.96 0.97 0.97 0.97 0.98 0.98 0.99 0.99	0.96 0.97 0.97 0.97 0.98 0.98 0.99 0.99	0.96 0.97 0.97 0.98 0.98 0.98 0.98 0.99 0.99	0.97 0.97 0.98 0.98 0.98 0.99 0.99 0.99	0.98 0.98 0.98 0.98 0.98 0.98 0.99 0.99	0.99 0.98 0.98 0.98 0.99 0.99 0.99 0.99	1.00 0.99 0.99 0.99 0.99 0.99 0.99 0.99	1.00 1.00 0.99 0.99 0.99 0.99 0.99 0.99	1.02 1.01 1.00 0.99 0.99 0.99 0.99 0.99 1.00	1.02 1.01 1.00 0.99 0.99 0.99 0.99 0.99 1.00	1.01 1.00 0.99 0.99 0.99 0.99 0.99 0.99	1.00 0.99 0.99 0.99 0.99 0.99 0.99 0.99	0.99 0.99 0.98 0.98 0.98 0.99 0.99 0.99	0.98 0.98 0.98 0.98 0.98 0.98 0.98 0.99 0.99	0.97 0.97 0.98 0.98 0.98 0.98 0.99 0.99 0.99	0.97 0.97 0.97 0.98 0.98 0.98 0.99 0.99	0.96 0.97 0.97 0.97 0.98 0.98 0.99 0.99	0.96 0.97 0.97 0.97 0.98 0.98 0.99 0.99	0.89 0.92 0.94 0.95 0.97 0.98 0.99 0.99	0.90 0.92 0.94 0.96 0.97 0.98 0.99 0.99	0.91 0.94 0.95 0.97 0.98 0.98 0.99 1.00	0.93 0.95 0.97 0.98 0.98 0.99 0.99 1.00	0.97 0.98 0.99 0.99 1.00 1.00 1.00	1.01 1.01 1.01 1.01 1.01 1.00 1.00 1.00	1.07 1.05 1.04 1.02 1.02 1.01 1.01 1.00	1.13 1.09 1.06 1.04 1.03 1.02 1.01 1.00	1.18 1.11 1.07 1.05 1.03 1.02 1.01 1.01	1.18 1.11 1.07 1.05 1.03 1.02 1.01 1.01	1.13 1.09 1.06 1.04 1.03 1.02 1.01 1.00	1.07 1.05 1.04 1.02 1.02 1.01 1.01 1.00	1.01 1.01 1.01 1.01 1.00 1.00 1.00	0.97 0.98 0.99 0.99 0.99 1.00 1.00 1.00	0.93 0.95 0.97 0.98 0.98 0.99 0.99 1.00	0.94 0.95 0.97 0.98 0.98 0.99 1.00	0.92 0.94 0.96 0.97 0.98 0.99 0.99	0.92 0.94 0.95 0.97 0.98 0.99 0.99
0.96 0.97 0.97 0.98 0.98 0.99 0.99 1.00	0.96 0.97 0.97 0.97 0.98 0.98 0.99 0.99 1.00	0.96 0.97 0.97 0.98 0.98 0.98 0.98 0.99 0.99 1.00	0.97 0.97 0.98 0.98 0.98 0.99 0.99 0.99 0.99 1.00	0.98 0.98 0.98 0.98 0.98 0.98 0.99 0.99	0.99 0.98 0.98 0.98 0.98 0.99 0.99 0.99	1.00 0.99 0.99 0.99 0.99 0.99 0.99 0.99	1.00 1.00 0.99 0.99 0.99 0.99 0.99 1.00 1.00	1.02 1.01 1.00 0.99 0.99 0.99 0.99 0.99 1.00 1.00	1.02 1.01 1.00 0.99 0.99 0.99 0.99 0.99 1.00 1.00	1.01 1.00 1.00 0.99 0.99 0.99 0.99 0.99	1.00 0.99 0.99 0.99 0.99 0.99 0.99 0.99	0.99 0.99 0.98 0.98 0.98 0.99 0.99 0.99	0.98 0.98 0.98 0.98 0.98 0.98 0.98 0.99 0.99	0.97 0.97 0.98 0.98 0.98 0.99 0.99 0.99 1.00	0.97 0.97 0.97 0.98 0.98 0.98 0.98 0.99 0.99 1.00	0.96 0.97 0.97 0.97 0.98 0.98 0.99 0.99 1.00	0.96 0.97 0.97 0.97 0.98 0.98 0.99 0.99 1.00	0.89 0.92 0.94 0.95 0.97 0.98 0.99 0.99 1.00	0.90 0.92 0.94 0.96 0.97 0.98 0.99 0.99 1.00	0.91 0.94 0.95 0.97 0.98 0.98 0.99 1.00 1.00	0.93 0.95 0.97 0.98 0.98 0.99 0.99 1.00 1.00	0.97 0.98 0.99 0.99 1.00 1.00 1.00 1.00	1.01 1.01 1.01 1.01 1.00 1.00 1.00 1.00	1.07 1.05 1.04 1.02 1.02 1.01 1.01 1.00 1.00	1.13 1.09 1.06 1.04 1.03 1.02 1.01 1.00 1.00	1.18 1.11 1.07 1.05 1.03 1.02 1.01 1.01 1.01	1.18 1.07 1.05 1.03 1.02 1.01 1.01 1.01	1.13 1.09 1.06 1.04 1.03 1.02 1.01 1.00 1.00	1.07 1.05 1.04 1.02 1.02 1.01 1.01 1.00 1.00	1.01 1.01 1.01 1.01 1.01 1.00 1.00 1.00	0.97 0.98 0.99 0.99 1.00 1.00 1.00 1.00	0.93 0.95 0.97 0.98 0.99 0.99 1.00 1.00	0.94 0.95 0.97 0.98 0.98 0.99 1.00 1.00	0.92 0.94 0.96 0.97 0.98 0.99 0.99 1.00	0.92 0.94 0.95 0.97 0.98 0.99 0.99 1.00

Each of these four squares contains just over 300 cells and represents a thin vertical slice through a honeycombed rock. The surface is at the top, and the source of moisture is spread uniformly across the bottom. This moisture makes its way up through the slice to the surface where it evaporates. The numerical value in each cell records the magnitude of the flow of moisture into and out of that cell. The spreadsheet also tracks the direction of the flow through each cell. The flow of water into and out of any one cell has to be the same because cells neither store nor supply water. The spreadsheet's task was to assign to cells a flow—magnitude and direction—that was commensurate with the flows in all four of its adjacent cells (up-down, left-right).

*Top left*. With a flawless surface, moisture rises uniformly from bottom to top. The flow through every cell is magnitude 1.00 and is shaded mid-grey.

All but top left. Holes at the surface of varying depths disrupt the uniform flow. Cells shaded white are carrying more than 1.05 units of flow. Cells shaded darkly are carrying less than 0.95 units of flow. The cells on the rim of the hole are most depleted of moisture, dropping from 1.00 to 0.84, 0.54, and 0.22 units as the depth of the hole increases. Correspondingly, evaporation from the bottom of the hole increases from 1.00 to 1.37, 1.63, and 2.05 units.

																																		_	
0.76	0.74	0.69	0.60	0.50			0.33	0.43	0.43	0.33			0.50	0.60	0.69	0.74	0.76	0.	28 0.2	5 0.21	0.14			0.10	0.14	0.17	0.17	0.14	0.10			0.14	0.21	0.25	0.28
0.78	0.77	0.75	0.75	0.98			0.52	0.57	0.57	0.52			0.98	0.75	0.75	0.77	0.78	0.	31 0.3	0.28	0.25			0.17	0.18	0.21	0.21	0.18	0.17			0.25	0.28	0.30	0.31
0.82	0.82	0.83	0.88	1.01	1.49	1.47	0.94	0.81	0.81	0.94	1.47	1.49	1.01	0.88	0.83	0.82	0.82	0.	38 0.3	9 0.40	0.41			0.27	0.26	0.28	0.28	0.26	0.27			0.41	0.40	0.39	0.38
0.86	0.87	0.89	0.94	1.04	1.20	1.19	1.03	0.95	0.95	1.03	1.19	1.20	1.04	0.94	0.89	0.87	0.86	0.	48 0.5	1 0.57	0.67			0.42	0.39	0.40	0.40	0.39	0.42			0.67	0.57	0.51	0.48
0.90	0.91	0.93	0.97	1.02	1.09	1.09	1.03	1.00	1.00	1.03	1.09	1.09	1.02	0.97	0.93	0.91	0.90	0.	59 0.6	4 0.76	1.16			0.62	0.57	0.56	0.56	0.57	0.62			1.16	0.76	0.64	0.59
0.92	0.93	0.95	0.97	1.01	1.04	1.04	1.03	1.01	1.01	1.03	1.04	1.04	1.01	0.97	0.95	0.93	0.92	0.	71 0.7	5 0.86	1.07	1.66	1.67	1.02	0.81	0.74	0.74	0.81	1.02	1.67	1.66	1.07	0.86	0.76	0.71
0.94	0.95	0.96	0.98	1.00	1.02	1.02	1.02	1.01	1.01	1.02	1.02	1.02	1.00	0.98	0.96	0.95	0.94	0.	81 0.8	5 0.93	1.07	1.28	1.28	1.07	0.93	0.87	0.87	0.93	1.07	1.28	1.28	1.07	0.93	0.85	0.81
0.95	0.96	0.97	0.98	0.99	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.00	0.99	0.98	0.97	0.96	0.95	0.	88 0.9	1 0.97	1.05	1.13	1.13	1.06	0.98	0.95	0.95	0.98	1.06	1.13	1.13	1.05	0.97	0.91	0.88
0.96	0.97	0.97	0.98	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.98	0.97	0.97	0.96	0.	93 0.9	5 0.98	1.03	1.07	1.07	1.04	1.01	0.98	0.98	1.01	1.04	1.07	1.07	1.03	0.98	0.95	0.93
0.97	0.97	0.97	0.98	0.99	0.99	0.99	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.98	0.97	0.97	0.97	0.	96 0.9	7 0.99	1.02	1.03	1.04	1.03	1.01	1.00	1.00	1.01	1.03	1.04	1.03	1.02	0.99	0.97	0.96
0.97	0.97	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.97	0.97	0.	98 0.9	8 0.99	1.01	1.02	1.02	1.02	1.01	1.01	1.01	1.01	1.02	1.02	1.02	1.01	0.99	0.98	0.98
0.98	0.98	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98	0.98	0.	99 0.9	9 1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	0.99	0.99
0.98	0.98	0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	0.98	0.98	0.	99 0.9	9 1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00	0.99	0.99
0.98	0.98	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.98	1.	00 1.0	0 1.00	1.00	1.00	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.01	1.00	1.00	1.00	1.00	1.00
0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	0.99	1.	00 1.0	0 1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.99	0.99	0.99	0.99	0.99	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99	1.	00 1.0	0 1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.	00 1.0	0 1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
0.37	0.34	0.30	0.23	0.16			0.06	0.08	0.08	0.06			0.16	0.23	0.30	0.34	0.37	0.	.45 0.4	4 0.40	0.34	0.26	0.18			0.01	0.01			0.18	0.26	0.34	0.40	0.44	0.45
0.37	0.34	0.30	0.23	0.16			0.06	0.08	0.08	0.06			0.16	0.23	0.30 0.35	0.34 0.38	0.37	0.	45 0.4 48 0.4	4 0.40 6 0.44	0.34	0.26 0.35	0.18 0.30			0.01 0.02	0.01 0.02			0.18 0.30	0.26 0.35	0.34 0.40	0.40 0.44	0.44 0.46	0.45 0.48
0.37 0.39 0.45	0.34 0.38 0.45	0.30 0.35 0.44	0.23 0.32 0.44	0.16 0.28 0.45			0.06 0.10 0.17	0.08 0.11 0.17	0.08 0.11 0.17	0.06 0.10 0.17			0.16 0.28 0.45	0.23 0.32 0.44	0.30 0.35 0.44	0.34 0.38 0.45	0.37 0.39 0.45	0. 0. 0.	.45 0.4 .48 0.4 .52 0.5	4 0.40 6 0.44 1 0.50	0.34 0.40 0.49	0.26 0.35 0.48	0.18 0.30 0.48			0.01 0.02 0.06	0.01 0.02 0.06			0.18 0.30 0.48	0.26 0.35 0.48	0.34 0.40 0.49	0.40 0.44 0.50	0.44 0.46 0.51	0.45 0.48 0.52
0.37 0.39 0.45 0.53	0.34 0.38 0.45 0.54	0.30 0.35 0.44 0.56	0.23 0.32 0.44 0.62	0.16 0.28 0.45 0.72			0.06 0.10 0.17 0.31	0.08 0.11 0.17 0.30	0.08 0.11 0.17 0.30	0.06 0.10 0.17 0.31			0.16 0.28 0.45 0.72	0.23 0.32 0.44 0.62	0.30 0.35 0.44 0.56	0.34 0.38 0.45 0.54	0.37 0.39 0.45 0.53	0. 0. 0.	45 0.4 48 0.4 52 0.5 58 0.5	4 0.40 6 0.44 1 0.50 8 0.58	0.34 0.40 0.49 0.60	0.26 0.35 0.48 0.66	0.18 0.30 0.48 0.76			0.01 0.02 0.06 0.15	0.01 0.02 0.06 0.15			0.18 0.30 0.48 0.76	0.26 0.35 0.48 0.66	0.34 0.40 0.49 0.60	0.40 0.44 0.50 0.58	0.44 0.46 0.51 0.58	0.45 0.48 0.52 0.58
0.37 0.39 0.45 0.53 0.62	0.34 0.38 0.45 0.54 0.64	0.30 0.35 0.44 0.56 0.68	0.23 0.32 0.44 0.62 0.81	0.16 0.28 0.45 0.72 1.21			0.06 0.10 0.17 0.31 0.53	0.08 0.11 0.17 0.30 0.53	0.08 0.11 0.17 0.30 0.53	0.06 0.10 0.17 0.31 0.53			0.16 0.28 0.45 0.72 1.21	0.23 0.32 0.44 0.62 0.81	0.30 0.35 0.44 0.56 0.68	0.34 0.38 0.45 0.54 0.64	0.37 0.39 0.45 0.53 0.62	0. 0. 0. 0.	45 0.4 48 0.4 52 0.5 58 0.5 64 0.6	4 0.40 6 0.44 1 0.50 8 0.58 5 0.61	0.34 0.40 0.49 0.60 7 0.72	0.26 0.35 0.48 0.66 0.85	0.18 0.30 0.48 0.76 1.26			0.01 0.02 0.06 0.15 0.40	0.01 0.02 0.06 0.15 0.40			0.18 0.30 0.48 0.76 1.26	0.26 0.35 0.48 0.66 0.85	0.34 0.40 0.49 0.60 0.72	0.40 0.44 0.50 0.58 0.67	0.44 0.46 0.51 0.58 0.65	0.45 0.48 0.52 0.58 0.64
0.37 0.39 0.45 0.53 0.62 0.71	0.34 0.38 0.45 0.54 0.64 0.74	0.30 0.35 0.44 0.56 0.68 0.79	0.23 0.32 0.44 0.62 0.81 0.90	0.16 0.28 0.45 0.72 1.21 1.10	1.68	1.63	0.06 0.10 0.17 0.31 0.53 1.01	0.08 0.11 0.17 0.30 0.53 0.84	0.08 0.11 0.17 0.30 0.53 0.84	0.06 0.10 0.17 0.31 0.53 1.01	1.63	1.68	0.16 0.28 0.45 0.72 1.21 1.10	0.23 0.32 0.44 0.62 0.81 0.90	0.30 0.35 0.44 0.56 0.68 0.79	0.34 0.38 0.45 0.54 0.64 0.74	0.37 0.39 0.45 0.53 0.62 0.71	0. 0. 0. 0. 0. 0.	45 0.4 48 0.4 52 0.5 58 0.5 .64 0.6 .71 0.7	4 0.40 6 0.44 1 0.50 8 0.58 5 0.61 3 0.76	0.34 0.40 0.49 0.60 7 0.72 0.82	0.26 0.35 0.48 0.66 0.85 0.93	0.18 0.30 0.48 0.76 1.26 1.13	1.69	1.57	0.01 0.02 0.06 0.15 0.40 1.05	0.01 0.02 0.06 0.15 0.40 1.05	1.57	1.69	0.18 0.30 0.48 0.76 1.26 1.13	0.26 0.35 0.48 0.66 0.85 0.93	0.34 0.40 0.49 0.60 0.72 0.82	0.40 0.44 0.50 0.58 0.67 0.76	0.44 0.46 0.51 0.58 0.65 0.73	0.45 0.48 0.52 0.58 0.64 0.71
0.37 0.39 0.45 0.53 0.62 0.71 0.79	0.34 0.38 0.45 0.54 0.64 0.74 0.82	0.30 0.35 0.44 0.56 0.68 0.79 0.87	0.23 0.32 0.44 0.62 0.81 0.90 0.96	0.16 0.28 0.45 0.72 1.21 1.10 1.10	1.68	1.63	0.06 0.10 0.17 0.31 0.53 1.01 1.10	0.08 0.11 0.17 0.30 0.53 0.84 0.99	0.08 0.11 0.17 0.30 0.53 0.84 0.99	0.06 0.10 0.17 0.31 0.53 1.01 1.10	1.63	1.68	0.16 0.28 0.45 0.72 1.21 1.10 1.10	0.23 0.32 0.44 0.62 0.81 0.90 0.96	0.30 0.35 0.44 0.56 0.68 0.79 0.87	0.34 0.38 0.45 0.54 0.64 0.74 0.82	0.37 0.39 0.45 0.53 0.62 0.71 0.79	0. 0. 0. 0. 0. 0. 0. 0.	45 0.4 48 0.4 52 0.5 58 0.5 64 0.6 71 0.7 78 0.7	4 0.40 6 0.44 1 0.50 8 0.58 5 0.67 3 0.76 9 0.83	0 0.34 0.40 0 0.49 0 0.60 7 0.72 0 0.82 0 0.88	0.26 0.35 0.48 0.66 0.85 0.93 0.97	0.18 0.30 0.48 0.76 1.26 1.13 1.12	1.69	1.57 1.31	0.01 0.02 0.06 0.15 0.40 1.05 1.18	0.01 0.02 0.06 0.15 0.40 1.05 1.18	1.57	1.69 1.32	0.18 0.30 0.48 0.76 1.26 1.13 1.12	0.26 0.35 0.48 0.66 0.85 0.93 0.97	0.34 0.40 0.49 0.60 0.72 0.82 0.88	0.40 0.44 0.50 0.58 0.67 0.76 0.83	0.44 0.46 0.51 0.58 0.65 0.73 0.79	0.45 0.48 0.52 0.58 0.64 0.71 0.78
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07	1.68 1.30 1.15	1.63 1.29 1.16	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05	0.06 0.10 0.17 0.31 0.53 1.01 1.10	1.63 1.29 1.16	1.68 1.30 1.15	0.16 0.28 0.45 1.21 1.10 1.10 1.07	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92	0.34 0.38 0.45 0.54 0.64 0.74 0.82	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86	0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	45 0.4 48 0.4 52 0.5 58 0.5 64 0.6 71 0.7 78 0.7 83 0.8	4 0.40 6 0.44 1 0.50 8 0.58 5 0.67 3 0.76 9 0.83 5 0.88	0 0.34 0 0.40 0 0.49 0 0.60 7 0.72 6 0.82 0.88 0.93	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09	1.69 1.32 1.18	1.57 1.31 1.20	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17	1.57 1.31 1.20	1.69 1.32 1.18	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00	0.34 0.40 0.49 0.60 0.72 0.82 0.88 0.93	0.40 0.44 0.50 0.58 0.67 0.76 0.83 0.88	0.44 0.46 0.51 0.58 0.65 0.73 0.79 0.85	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88 0.93	0.30 0.35 0.44 0.56 0.68 0.79 0.92 0.92	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05	1.68 1.30 1.15 1.09	1.63 1.29 1.16 1.10	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.08	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06	0.06 0.10 0.17 0.31 1.01 1.10 1.10 1.08	1.63 1.29 1.16 1.10	1.68 1.30 1.15 1.09	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88 0.93	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.	.45         0.4           .48         0.4           .52         0.5           .58         0.5           .64         0.6           .71         0.7           .78         0.7           .83         0.8	4 0.40 6 0.44 1 0.50 8 0.58 5 0.67 3 0.76 9 0.83 5 0.88	0 0.34 0.40 0 0.49 0 0.60 7 0.72 0 0.82 0.88 0.93 0.95	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06	1.69 1.32 1.18 1.11	1.57 1.31 1.20 1.13	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14	1.57 1.31 1.20 1.13	1.69 1.32 1.18 1.11	0.18 0.30 0.48 1.26 1.13 1.12 1.09 1.06	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00	0.34 0.40 0.49 0.60 0.72 0.82 0.88 0.93 0.95	0.40 0.44 0.50 0.58 0.67 0.76 0.83 0.88 0.91	0.44 0.46 0.51 0.58 0.65 0.73 0.79 0.85 0.89	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83 0.88
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88 0.93 0.96	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.92 0.96	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04	1.68 1.30 1.15 1.09 1.06	1.63 1.29 1.16 1.10	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.08 1.07	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06	0.06 0.10 0.17 0.31 1.01 1.10 1.10 1.08 1.07	1.63 1.29 1.16 1.10 1.07	1.68 1.30 1.15 1.09 1.06	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.92 0.98	0.34 0.38 0.45 0.54 0.54 0.64 0.82 0.88 0.93 0.96	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	45 0.4 48 0.4 52 0.5 58 0.5 58 0.5 64 0.6 71 0.7 78 0.7 78 0.7 83 0.8 88 0.8 91 0.9	4 0.40 6 0.44 1 0.50 8 0.58 5 0.61 3 0.76 9 0.83 5 0.88 9 0.91 2 0.94	0.34 0.40 0.49 0.60 7 0.72 0.82 0.88 0.93 0.93 0.95	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04	1.69 1.32 1.18 1.11 1.07	1.57 1.31 1.20 1.13 1.09	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14 1.10	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14 1.10	1.57 1.31 1.20 1.13 1.09	1.69 1.32 1.18 1.11 1.07	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00	0.34 0.40 0.49 0.60 0.72 0.82 0.88 0.93 0.95 0.97	0.40 0.44 0.50 0.58 0.67 0.76 0.83 0.83 0.91 0.94	0.44 0.46 0.51 0.58 0.65 0.73 0.79 0.85 0.89 0.92	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83 0.83 0.83
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88 0.93 0.96 0.98	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.98	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03	1.68 1.30 1.15 1.09 1.06 1.04	1.63 1.29 1.16 1.10 1.07 1.05	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.08 1.07 1.05	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.06	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.06 1.05	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.08 1.07 1.05	1.63 1.29 1.16 1.10 1.07 1.05	1.68 1.30 1.15 1.09 1.06 1.04	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.99	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88 0.93 0.96 0.98	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	45 0.4 48 0.4 52 0.5 58 0.5 64 0.6 71 0.7 78 0.7 78 0.7 83 0.8 88 0.8 91 0.9 94 0.9	4 0.40 6 0.44 1 0.50 8 0.58 5 0.61 3 0.76 9 0.81 5 0.88 9 0.91 2 0.94 4 0.96	0.34 0.40 0.49 0.60 7 0.72 0.82 0.88 0.93 0.95 0.95 0.98	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00 1.00	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04 1.03	1.69 1.32 1.18 1.11 1.07 1.05	1.57 1.31 1.20 1.13 1.09 1.07	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14 1.10 1.07	0.01 0.02 0.06 0.15 1.05 1.18 1.17 1.14 1.10 1.07	1.57 1.31 1.20 1.13 1.09 1.07	1.69 1.32 1.18 1.11 1.07 1.05	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04 1.03	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00 1.00	0.34 0.40 0.49 0.60 0.72 0.82 0.88 0.93 0.95 0.97 0.98	0.40 0.44 0.50 0.58 0.67 0.83 0.88 0.91 0.94 0.96	0.44 0.46 0.51 0.58 0.73 0.73 0.85 0.89 0.92 0.94	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83 0.88 0.91 0.91
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99	0.34 0.38 0.45 0.54 0.64 0.82 0.88 0.93 0.96 0.98 0.99	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.99 0.99 1.00	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03 1.02	1.68 1.30 1.15 1.09 1.06 1.04 1.03	1.63 1.29 1.16 1.10 1.07 1.05 1.04	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.08 1.07 1.05 1.04	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.06 1.05 1.04	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.06 1.05 1.04	0.06 0.10 0.17 0.31 1.01 1.10 1.10 1.08 1.07 1.05 1.04	1.63 1.29 1.16 1.10 1.07 1.05 1.04	1.68 1.30 1.15 1.09 1.06 1.04 1.03	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03 1.02	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.99 1.00	0.34 0.38 0.45 0.54 0.54 0.64 0.82 0.82 0.88 0.93 0.96 0.98 0.99	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	45 0.4 48 0.4 52 0.5 58 0.5 64 0.6 71 0.7 78 0.7 78 0.7 78 0.7 9.0 9.0 9.0 9.0 9.0 9.0 9.0 9.0 9.0 9.0	4 0.40 6 0.44 1 0.50 8 0.58 5 0.61 3 0.76 9 0.81 5 0.88 9 0.91 2 0.94 4 0.96 6 0.97	0.34 0.40 0.49 0.60 0.72 0.72 0.82 0.88 0.93 0.93 0.95 0.97 0.98 0.99	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00 1.00 1.00	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04 1.03 1.02	1.69 1.32 1.18 1.11 1.07 1.05 1.04	1.57 1.31 1.20 1.13 1.09 1.07 1.05	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14 1.10 1.07 1.05	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14 1.10 1.07 1.05	1.57 1.31 1.20 1.13 1.09 1.07 1.05	1.69 1.32 1.18 1.11 1.07 1.05 1.04	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04 1.03 1.02	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00 1.00 1.00	0.34 0.40 0.49 0.60 0.72 0.82 0.88 0.93 0.95 0.97 0.98 0.99	0.40 0.44 0.50 0.58 0.67 0.83 0.88 0.91 0.94 0.96 0.97	0.44 0.46 0.51 0.58 0.65 0.73 0.79 0.85 0.89 0.92 0.94 0.96	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83 0.83 0.91 0.94 0.96
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88 0.93 0.96 0.98 0.99 1.00	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.99 1.00	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01 1.01	0.16 0.28 0.45 0.72 1.21 1.10 1.07 1.07 1.07 1.05 1.04 1.03 1.02	1.68 1.30 1.15 1.09 1.06 1.04 1.03 1.03	1.63 1.29 1.16 1.10 1.07 1.05 1.04 1.03	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.08 1.07 1.05 1.04 1.03	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.06 1.05 1.05 1.04 1.03	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.05 1.05 1.05 1.04 1.03	0.06 0.10 0.17 0.31 1.01 1.10 1.00 1.08 1.07 1.05 1.04 1.03	1.63 1.29 1.16 1.10 1.07 1.05 1.04 1.03	1.68 1.30 1.15 1.09 1.06 1.04 1.03	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03 1.02	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01 1.01	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.99 1.00 1.00	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.93 0.93 0.96 0.98 0.99 1.00	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	45 0.4 48 0.4 52 0.5 58 0.5 58 0.5 64 0.6 4 0.6 71 0.7 78 0.7 78 0.7 78 0.7 78 0.9 91 0.9 94 0.9 96 0.9 97 0.9	4 0.40 6 0.44 1 0.50 8 0.58 5 0.67 3 0.76 9 0.83 9 0.91 2 0.94 4 0.96 6 0.97 7 0.98	<ul> <li>0.34</li> <li>0.40</li> <li>0.49</li> <li>0.60</li> <li>0.72</li> <li>0.82</li> <li>0.88</li> <li>0.93</li> <li>0.95</li> <li>0.97</li> <li>0.98</li> <li>0.99</li> <li>0.99</li> </ul>	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00 1.00 1.00 1.00	0.18 0.30 0.48 0.76 1.25 1.13 1.12 1.09 1.06 1.04 1.03 1.02 1.01	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03	0.01 0.02 0.06 0.15 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04	0.01 0.02 0.06 0.40 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04 1.03 1.02 1.01	0.26 0.35 0.48 0.66 0.85 0.93 0.97 1.00 1.00 1.00 1.00 1.00	0.34 0.40 0.49 0.60 0.72 0.82 0.88 0.93 0.95 0.97 0.98 0.99 0.99	0.40 0.44 0.50 0.58 0.76 0.83 0.83 0.91 0.91 0.94 0.96 0.97 0.98	0.44 0.46 0.51 0.58 0.65 0.73 0.79 0.85 0.89 0.92 0.94 0.94 0.96 0.97	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83 0.83 0.91 0.91 0.94 0.96 0.97
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88 0.93 0.96 0.98 0.99 1.00 1.00	0.30 0.35 0.44 0.56 0.68 0.79 0.92 0.92 0.96 0.98 0.99 1.00 1.00	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01 1.01 1.01	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03 1.02 1.02 1.01	1.68 1.30 1.15 1.09 1.06 1.04 1.03 1.03 1.03	1.63 1.29 1.16 1.10 1.07 1.05 1.04 1.03 1.02	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.08 1.07 1.05 1.04 1.03 1.02	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.06 1.06 1.05 1.04 1.03 1.02	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.05 1.06 1.05 1.04 1.03 1.02	0.06 0.10 0.17 0.31 1.01 1.10 1.00 1.08 1.07 1.05 1.04 1.03 1.02	1.63 1.29 1.16 1.10 1.07 1.05 1.04 1.03 1.02	1.68 1.30 1.15 1.09 1.06 1.04 1.03 1.03 1.02	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03 1.02 1.02 1.01	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01 1.01 1.01	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.99 1.00 1.00 1.01	0.34 0.38 0.45 0.54 0.64 0.82 0.88 0.93 0.96 0.98 0.99 1.00 1.00	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00 1.00	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	45 0.4 48 0.4 52 0.5 58 0.5 64 0.6 71 0.7 78 0.7 78 0.7 78 0.8 88 0.8 88 0.8 91 0.9 91 0.9 96 0.9 97 0.9 98 0.9	4 0.40 6 0.44 1 0.50 8 0.58 5 0.67 3 0.76 9 0.83 5 0.83 9 0.91 2 0.94 4 0.96 6 0.97 7 0.98 8 0.99	<ul> <li>0.34</li> <li>0.40</li> <li>0.49</li> <li>0.60</li> <li>0.82</li> <li>0.82</li> <li>0.83</li> <li>0.93</li> <li>0.95</li> <li>0.97</li> <li>0.99</li> <li>0.99</li> </ul>	0.26 0.35 0.48 0.66 0.85 0.93 1.00 1.00 1.00 1.00 1.00 1.00 1.00	0.18 0.30 0.48 0.76 1.26 1.13 1.09 1.06 1.04 1.03 1.02 1.01	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02 1.02	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03 1.02	0.01 0.02 0.06 0.15 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04 1.02	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04 1.02	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03 1.02	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02 1.02	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04 1.03 1.02 1.01 1.01	0.26 0.35 0.48 0.66 0.85 0.97 1.00 1.00 1.00 1.00 1.00 1.00 1.00	0.34 0.40 0.49 0.60 0.72 0.82 0.93 0.93 0.95 0.97 0.99 0.99	0.40 0.44 0.50 0.58 0.67 0.76 0.83 0.91 0.94 0.94 0.94 0.95	0.44 0.46 0.51 0.58 0.65 0.73 0.85 0.89 0.92 0.94 0.94 0.96 0.97 0.98	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83 0.88 0.91 0.94 0.96 0.97 0.98
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00 1.00	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88 0.93 0.96 0.98 0.99 1.00 1.00	<ul> <li>0.30</li> <li>0.35</li> <li>0.44</li> <li>0.56</li> <li>0.68</li> <li>0.79</li> <li>0.92</li> <li>0.96</li> <li>0.98</li> <li>0.99</li> <li>1.00</li> <li>1.00</li> <li>1.01</li> </ul>	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01 1.01 1.01 1.01	0.16 0.28 0.45 0.72 1.21 1.10 1.07 1.05 1.04 1.03 1.02 1.02 1.01	1.68 1.30 1.15 1.09 1.06 1.04 1.03 1.03 1.03 1.02 1.01	1.63 1.29 1.16 1.10 1.05 1.04 1.03 1.02 1.01	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.08 1.07 1.05 1.04 1.03 1.02 1.01	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.05 1.06 1.05 1.04 1.03 1.02 1.01	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.05 1.06 1.05 1.04 1.03 1.02 1.01	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.01 1.08 1.07 1.05 1.04 1.03 1.02 1.01	1.63 1.29 1.16 1.10 1.07 1.05 1.04 1.03 1.02 1.01	1.68 1.30 1.15 1.09 1.06 1.04 1.03 1.03 1.03 1.02 1.01	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03 1.02 1.02 1.01	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01 1.01 1.01 1.01	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.99 1.00 1.00 1.01	0.34 0.38 0.45 0.54 0.54 0.54 0.54 0.54 0.82 0.93 0.93 0.93 0.99 1.00 1.00 1.01	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00 1.00	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	45 0.4 48 0.4 52 0.5 58 0.5 64 0.6 71 0.7 78 0.7 78 0.7 78 0.7 78 0.9 91 0.9 94 0.9 95 0.9 99 0.9	4 0.40 6 0.44 1 0.50 8 0.58 5 0.67 3 0.76 9 0.83 5 0.88 9 0.91 2 0.94 4 0.96 6 0.97 7 0.98 8 0.95 9 0.95	0.34           0.40           0.49           0.93           0.94           0.99           0.99           0.99           0.99	0.26 0.35 0.48 0.66 0.93 0.93 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.0	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04 1.03 1.02 1.01 1.01	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02 1.02 1.02	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03 1.02 1.01	0.01 0.02 0.06 1.05 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04 1.02 1.01	0.01 0.02 0.06 0.15 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04 1.02 1.01	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03 1.02 1.01	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02 1.02 1.02	0.18 0.30 0.48 0.76 1.26 1.13 1.09 1.06 1.04 1.03 1.02 1.01 1.01 1.01	0.26 0.35 0.66 0.93 0.93 1.00 1.00 1.00 1.00 1.00 1.00 1.00	0.34 0.40 0.49 0.60 0.72 0.82 0.93 0.93 0.95 0.97 0.99 0.99 1.00	0.40 0.44 0.50 0.58 0.67 0.83 0.83 0.91 0.94 0.94 0.94 0.97 0.98 0.99	0.44 0.46 0.51 0.58 0.65 0.73 0.79 0.85 0.89 0.92 0.92 0.94 0.96 0.97 0.98 0.99	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83 0.88 0.91 0.94 0.94 0.94 0.95 0.99
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00 1.00 1.00	0.34 0.38 0.45 0.54 0.54 0.74 0.82 0.88 0.93 0.96 0.98 0.99 1.00 1.00 1.00	<ul> <li>0.30</li> <li>0.35</li> <li>0.44</li> <li>0.56</li> <li>0.68</li> <li>0.79</li> <li>0.92</li> <li>0.96</li> <li>0.98</li> <li>0.99</li> <li>1.00</li> <li>1.01</li> <li>1.01</li> <li>1.00</li> </ul>	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01 1.01 1.01 1.01 1.01 1.0	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.05 1.04 1.03 1.02 1.02 1.01 1.01	1.68 1.30 1.15 1.09 1.06 1.04 1.03 1.03 1.03 1.02 1.01 1.00	1.63 1.29 1.16 1.10 1.07 1.05 1.04 1.03 1.02 1.01 1.01	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.10 1.08 1.07 1.05 1.04 1.03 1.02 1.01	0.08 0.11 0.30 0.53 0.84 0.99 1.05 1.05 1.06 1.06 1.05 1.04 1.03 1.02 1.01	0.08 0.11 0.30 0.53 0.84 0.99 1.05 1.06 1.06 1.06 1.06 1.04 1.03 1.02 1.01	0.06 0.10 0.31 0.31 1.01 1.00 1.03 1.07 1.05 1.04 1.03 1.02 1.01	1.63 1.29 1.16 1.10 1.07 1.05 1.04 1.03 1.02 1.01	1.68 1.30 1.15 1.09 1.06 1.04 1.03 1.03 1.03 1.02 1.01 1.00	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.05 1.04 1.03 1.02 1.02 1.01 1.01	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01 1.01 1.01 1.01 1.01	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.99 1.00 1.00 1.01 1.01	0.34 0.38 0.45 0.54 0.54 0.82 0.88 0.93 0.96 0.98 0.99 1.00 1.00 1.01	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00 1.00 1.00	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	45 0.4 48 0.4 52 0.5 58 0.5 64 0.6 64 0.6 71 0.7 78 0.7 83 0.8 88 0.8 88 0.8 91 0.9 94 0.9 95 0.9 99 0.9 99 0.9 99 1.0	4 0.40 6 0.44 1 0.50 8 0.58 5 0.67 3 0.76 9 0.83 5 0.88 9 0.91 2 0.94 4 0.96 6 0.97 7 0.98 8 0.99 9 0.95 0 1.00	<ul> <li>0.34</li> <li>0.40</li> <li>0.49</li> <li>0.60</li> <li>0.72</li> <li>0.82</li> <li>0.88</li> <li>0.93</li> <li>0.93</li> <li>0.95</li> <li>0.97</li> <li>0.98</li> <li>0.99</li> <li>0.99</li> <li>0.99</li> <li>1.00</li> </ul>	0.26 0.35 0.48 0.85 0.93 0.97 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.0	0.18 0.30 0.48 1.26 1.26 1.13 1.12 1.09 1.06 1.04 1.03 1.02 1.01 1.01 1.01 1.01	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02 1.02 1.01 1.00	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03 1.02 1.01 1.01	0.01 0.02 0.06 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04 1.02 1.01 1.01	0.01 0.02 0.06 0.15 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04 1.02 1.01	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03 1.02 1.01 1.01	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02 1.02 1.01 1.00	0.18 0.30 0.48 1.26 1.26 1.13 1.09 1.06 1.04 1.03 1.02 1.01 1.01 1.01 1.01	0.26 0.35 0.48 0.85 0.93 0.97 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.0	0.34 0.40 0.49 0.60 0.72 0.82 0.93 0.93 0.95 0.99 0.99 1.00 1.00	0.40 0.44 0.50 0.58 0.67 0.83 0.83 0.88 0.91 0.94 0.96 0.97 0.98 0.99 1.00	0.44 0.46 0.51 0.58 0.65 0.73 0.79 0.85 0.89 0.92 0.94 0.96 0.97 0.98 0.99 1.00	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83 0.88 0.91 0.94 0.96 0.97 0.98 0.99
0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00 1.00 1.00 1.00	0.34 0.38 0.45 0.54 0.64 0.74 0.82 0.88 0.93 0.96 0.98 0.99 1.00 1.00 1.01 1.00	0.30 0.35 0.44 0.56 0.68 0.79 0.92 0.96 0.99 1.00 1.01 1.01 1.01 1.01 1.00	0.23 0.32 0.44 0.62 0.81 0.90 0.96 1.00 1.01 1.01 1.01 1.01 1.01 1.01 1.0	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03 1.02 1.01 1.01 1.01 1.01 1.01	1.68 1.30 1.15 1.09 1.06 1.04 1.03 1.03 1.02 1.01 1.00 1.00	1.63 1.29 1.16 1.10 1.05 1.04 1.03 1.02 1.01 1.01 1.01	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.10 1.10 1.08 1.07 1.05 1.04 1.03 1.02 1.01 1.01 1.01	0.08 0.11 0.30 0.53 0.84 0.99 1.05 1.06 1.06 1.06 1.06 1.05 1.04 1.03 1.02 1.01 1.01	0.08 0.11 0.17 0.30 0.53 0.84 0.99 1.05 1.06 1.05 1.06 1.05 1.04 1.03 1.02 1.01 1.01 1.01	0.06 0.10 0.17 0.31 0.53 1.01 1.10 1.00 1.08 1.07 1.05 1.04 1.03 1.02 1.01 1.01 1.00	1.63 1.29 1.16 1.10 1.07 1.05 1.04 1.03 1.02 1.01 1.01 1.01 1.01	1.68 1.30 1.15 1.09 1.06 1.04 1.03 1.02 1.01 1.00 1.00	0.16 0.28 0.45 0.72 1.21 1.10 1.10 1.07 1.05 1.04 1.03 1.02 1.01 1.01 1.01 1.01 1.01	0.23 0.32 0.44 0.62 0.81 0.90 0.96 0.98 1.00 1.01 1.01 1.01 1.01 1.01 1.01 1.0	0.30 0.35 0.44 0.56 0.68 0.79 0.87 0.92 0.96 0.98 0.99 1.00 1.00 1.01 1.01 1.01	0.34 0.38 0.45 0.54 0.64 0.82 0.88 0.93 0.96 0.98 0.99 1.00 1.00 1.01 1.00	0.37 0.39 0.45 0.53 0.62 0.71 0.79 0.86 0.91 0.95 0.97 0.99 1.00 1.00 1.00 1.00	0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0	45         0.4           48         0.4           52         0.5           58         0.5           64         0.6           67         0.7           78         0.7           78         0.7           78         0.7           78         0.7           79         0.9           91         0.9           92         0.9           93         0.9           94         0.9           97         0.9           98         0.9           99         0.9           99         1.0           0.0         1.0	4 0.40 6 0.44 1 0.50 8 0.58 5 0.61 3 0.76 9 0.83 5 0.88 9 0.91 2 0.94 4 0.96 6 0.97 7 0.98 8 0.99 9 0.99 9 0.99 0 1.00 0 1.00	<ul> <li>0.34</li> <li>0.40</li> <li>0.49</li> <li>0.60</li> <li>0.72</li> <li>0.82</li> <li>0.88</li> <li>0.93</li> <li>0.95</li> <li>0.95</li> <li>0.98</li> <li>0.97</li> <li>0.98</li> <li>0.99</li> <li>0.99</li> <li>0.99</li> <li>0.99</li> <li>0.99</li> <li>1.00</li> <li>1.00</li> </ul>	0.26 0.35 0.48 0.85 0.93 0.97 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.0	0.18 0.30 0.48 0.76 1.26 1.13 1.12 1.09 1.06 1.04 1.03 1.02 1.01 1.01 1.01 1.00 1.00	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02 1.02 1.02 1.01 1.00 1.00	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03 1.02 1.01 1.01 1.01 1.01	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04 1.02 1.01 1.01 1.01	0.01 0.02 0.06 0.15 0.40 1.05 1.18 1.17 1.14 1.10 1.07 1.05 1.04 1.02 1.01 1.01 1.01	1.57 1.31 1.20 1.13 1.09 1.07 1.05 1.03 1.02 1.01 1.01 1.01	1.69 1.32 1.18 1.11 1.07 1.05 1.04 1.02 1.02 1.02 1.01 1.00 1.00	0.18 0.30 0.48 1.26 1.26 1.13 1.09 1.04 1.04 1.04 1.02 1.01 1.01 1.01 1.00 1.00	0.26 0.35 0.48 0.85 0.93 0.97 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.0	0.34 0.40 0.49 0.60 0.72 0.82 0.93 0.93 0.95 0.97 0.98 0.99 0.99 1.00 1.00 1.00	0.40 0.44 0.50 0.58 0.67 0.76 0.83 0.88 0.91 0.94 0.96 0.97 0.98 0.99 0.99 1.00	0.44 0.46 0.51 0.58 0.65 0.73 0.79 0.85 0.89 0.92 0.94 0.96 0.97 0.98 0.99 1.00	0.45 0.48 0.52 0.58 0.64 0.71 0.78 0.83 0.83 0.91 0.94 0.96 0.97 0.98 0.99 0.99 1.00

Another four squares containing just over 300 cells representing a thin vertical slice through a honeycombed rock. This time there are two holes represented.

*Top row*: When the holes are well-separated or shallow, there's relatively little interaction between the holes.

*Bottom row*: Holes closer together or deeper begin to act as a single hole. The cells at the surface mid-way between the holes are severely depleted of moisture. In the square *bottom left*, the surface value on the wall is 0.08 units, and in the square *bottom right* only 0.01 units. Evaporation from the surface of the wall has practically ceased, which must be why in real life, walls endure.



The sought for numerical relationships obtained from a series of spreadsheet scenarios are summarized in the graph above. It shows the reduction of evaporation at the surface (vertical scale) as a function of distance from a hole (horizontal scale) for various depths of hole.

Very conveniently, the data in this graph can be fairly accurately summarized as:

 $\frac{1}{1+2(D-0.5)d^{-1.4}}$  where:

D is the distance at the surface from the centre of the hole.<sup>9</sup> Distances are reckoned as a multiple or submultiple of the diameter of the hole; and

d is the depth of the hole, again reckoned as a multiple or submultiple of the diameter of the hole.

The formula gives the value of the depletion at the surface. For D=0.5 for example, corresponding to the rim of the hole, the

depletion is 1.0—there is no evaporation from this particular point on the surface. The depletion drops as D gets larger, and for any given distance "D", the depletion increases as the depth "d" increases.

The test of any scientific theory is its ability to make predictions. And, I think this one can. Back in September 2003, according to my field notebook, I made some measurements, I forget why, of the diameter and pitches of various rows of honeycomb holes.

My notes say I made measurements of eighty such holes, which statistically isn't many, but then this is not a Ph.D. thesis so it will have to do. What was remarkable about



these measurements was that they showed that the pitch-to-diameter ratio of rows of smallish round honeycomb holes was astonishingly constant. No matter what their size, the pitch was always about 1.6 times their diameter. As I remarked in a photograph caption earlier, even the inland-

 $<sup>^9</sup>$  Values of D < 0.5 are not valid because this is within the hole.

side caverns called the Malaspina Galleries on Gabriola show close to this ratio. I had no idea at the time why this was so. Now, more than ten years later, I can make an educated guess.

According to my spreadsheet-derived formula, the only way that the interference between one hole and an adjacent hole can be proportional only to their diameters is if the ratio of the depths of the holes to their diameters is constant. In other words, in my formula, if that awkward looking  $d^{-1.4}$  is always the same.<sup>10</sup> The interference by one hole on the next is then only dependent on D, the ratio of their distance apart to their diameters. Which is what I had observed.

So are the depths of honeycomb holes a constant proportion of their diameters? A quick visit to the beach confirmed that they are. As long as the holes are not in a case-hardened surface—they have no overhanging rims—and as long as the holes are not old and being worn down, the depths of honeycomb holes tend to be equal to their diameters.

I actually only measured about thirty holes, but I stopped poking my skewer into holes to measure their depth when the measurements became predictably repetitive. Measure the depth, and you have the diameter and this is a consequence of shading of the floor of the hole by its walls (see Appendix 1 for details).



*Top*: observations showing pitch/diameter ratios for rows of nearly-equal-sized holes with various diameters.

*Middle*: observations showing depths of holes almost equal their diameters.

*Bottom*: spreadsheet prediction of surface depletion assuming depth and diameter are equal.

<sup>&</sup>lt;sup>10</sup> Better yet, if "d" is close to 1 as it appears to nearly always be, then  $d^{-1.4}$  is also close to 1. You can square, cube, square root, cube root, reciprocate... do whatever you like to 1 and the answer is always 1.

#### The tray of coins model

One very simple way to investigate how equally-sized circles arrange themselves on a plane is to use coins. Those on the tray shown *right* are all nickels. They were placed on the tray at random and the tray was then gently tilted until all the coins had made contact. Note that the coins are *not* meant to represent honeycomb holes directly, rather they are meant to represent the "zones of influence" of honeycomb holes. I had to assume these are circular because I was told by the bank that polygon-shaped Canadian coins are rare. This is an important point I'll come back to a bit later.



What can you do with such a tray of coins? Well first off, you can look for coins in lines, especially parallel lines. There are some, but my impression is that they are definitely not as obvious as on honeycomb surfaces—the lines of coins tend to wobble.

Then you look for patterns. Here are some:

- 3-coin arrays (A: triangles, D3=3A's: filled in hexagons)
- 4-coin arrays (B: squares, B1: skewed squares)
- 5-coin arrays (C: pentagons, C1 pentagons with a right angle), and

6-coin arrays (D: hexagons (rare), D1: pyramids, and D2: buckled rows).



Classifying honeycomb holes this way is obviously a subjective process as the "zones of influence" are not demarked in the way that the edges of coins are, but all of these patterns and a few more can be found. Examples of polygons are shown below.

When looking for these arrangements, I found it sometimes easier to concentrate on the pattern of the walls of the holes rather than the holes themselves. Triangles have a Y; squares have a +, and so on.











D



D3



Below: A network analysis of the array of coins in the photograph on the previous page.

Left: Another network analysis of an array of honeycomb holes. The array itself is shown bottom left.

This analysis method is not completely objective so you may disagree with some details.





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The next thing I did—I wasn't sure at this point on how to procede—was to mark the centres of the coins and then connect the centres into a network. In doing this, I was guided by what I perceived to be parallel lines of coins. The result you see on the bottom righthand side of the previous page.

I did this for three more arrays. One was an array of honeycomb holes. The second was another coin array but with some nickels replaced by quarters and dimes. The third array was an array of random dots created by the computer. For this final array there was no restriction on where the dots were placed—they had, in effect, no zones of influence.

Having done all this, for each network, I counted the number of 3-sided, 4sided, and so on polygons. I'll give the results in tabular form, but they are far easier seen in the accompanying ternary diagram.

	3-	4-	5-	6-	7-
rock	17%	64%	15%	4%	0%
nickels	31%	51%	13%	5%	0%
coins mixed	42%	38%	18%	1%	1%
random	82%	16%	2%	0%	0%

Some of the conclusions one can draw are:

(a) the array of honeycomb holes is very unlike a random array

(b) the array of honeycomb holes most closely resembles an array of equally-sized coins

(c) the array of honeycomb holes has a higher proportion of 4-sided polygons than any other; and

(d) hexagon arrangements are relatively rare in all the arrays.



A ternary diagram—the results of the network analyses. The positions of the data points within the triangle indicate the relative proportions of 3-sided polygons (top), 4-sided polygons (bottom left), and 5or more-sided polygons (bottom right) in the four networks.



### Hexagons. What hexagons?

We're now ready to tackle the question of what happens to honeycombing when it reaches a stage beyond being arrays of small, round holes.

So far, we have assumed that the "zones of influence" of holes are circular, but as the holes increase their diameters and increase their depths, there comes a time when zones of influence come into contact with each other. What then?

I would contend that what happens is that a thin wall is left where the zones are in contact because both holes are starving it of moisture and hence salt, and that from the perspective of the interior of the rock, the two holes in intimate contact then behave as one. It's like blowing up balloons in a box. They start round, but eventually they squeeze each other flat as they equalize their internal pressures (see Appendix 5 for what might be a biological example of this).

When this process is complete, and every zone of influence is in contact with an adjacent zone of influence, and there is nowhere left on the surface that is not within a zone of influence, what is left is a network of walls, and the geometry of these walls is defined by a set of polygons.

These sets of polygons are known to mathematicians as Voronoi diagrams, but are more familiar to geographers and earth scientists as Thiessen polygons, so that's what I'll call them. Thiesssen polygons have many applications, to which we are now going to add another.

The basic idea of a Thiessen polygon is pretty simple. Given a *plane*—a white picnic tablecloth for example—and a number of scattered *points* on that plane let's say squashed ants—then Thiessen polygons completely cover the tablecloth, they tile it, according to the following rules:

(a) every polygon contains one, and only one, squashed ant; and

(b) everywhere within a polygon, the distance to the squashed ant associated with that polygon is less than the distance to any other squashed ant on the table.

It follows from these specifications that Thiessen polygons are constructed of



A white picnic tablecloth and some squashed ants. Each ant is associated with one polygon that is defined so that everywhere within that polygon is closer to that ant than it is to any other ant.

straight lines along which the two nearest squashed ants are equidistant.

In the analysis of the arrangements of coins described earlier, the simplest array was of three coins (A) in a triangle, and in the network analysis it was counted as such.



In the Thiessen-network analysis we are now going to use instead, the 3-sided array A is depicted as a node, which is equidistant from all three coins, and 3 boundaries, each equidistant from 2 of the coins.



Similarly, the 4-sided array B (a square) is now depicted as a node, which is equidistant from all four coins, plus 4 boundaries, again each of which is equidistant from 2 of the coins.



The 5-sided array C (a pentagon) is now depicted as a node which is equidistant from all five coins plus 5 boundaries.



And the 6-sided array C (a hexagon) is now depicted as a node which is equidistant from all six coins plus 6 boundaries.

As already noted, in most practical situations, including on cliff faces, such geometrically-regular arrays are not usual. Irregular arrays are far more common and these require more than one node in their Thiessen diagram, and all the nodes are equidistant from the three nearest coins.<sup>11</sup> For example:





One very important improvement in the Thiessen-polygon method of analysis of arrays of holes is that it is objective. If several people do it, they arrive at the same answer. Computers are commonly used to generate Thiessen diagrams with no human intervention. This cannot be said of the "joining the dots" approach to network analysis I have used hitherto.

In an earlier article on honeycombing,<sup>3</sup> I was identifying holes as providing a service to the sandstone in that they allow internal moisture to escape and I wrote:

...how do you distribute *service centres* evenly over a flat surface that extends indefinitely in all directions? This task confronts many people, including those who decide where to locate schools, pubs, Canadian Tire stores—that sort of thing especially on prairies. A very popular way of distributing centres is to assign each of them to a hexagonal area.... What's nice about this pattern is that you can extend the coverage as far as you like in any direction, leaving everyone within easy reach of one of the centres.

While this is all true, it overlooks the fact that not many people are confronted with the

<sup>&</sup>lt;sup>11</sup> There are also often more boundaries than there are connections in the point-to-point network, which indicates that the point-to-point networks are often incomplete.

task of deciding where to locate large numbers of service centres on prairies all in one go. A more realistic problem is, given the scattered distribution of *existing* service centres, which may be where they are for various historical and other reasons, where is the best place to locate the next service centre? And one answer is, forget hexagons, and instead construct Thiessen polygons around the existing centres. The best place for the next centre is then within the polygon with the largest area. That way you split the polygon in two, and achieve the most beneficial reduction in distances people have to travel to a centre.

This must be what the honeycomb holes do. Initially, the holes do not interact, but as they grow in size, they take command, as it were, of an increasingly large proportion of the surface. This encourages holes to grow in areas that are least influenced by an existing hole, and hence the by-now highly interacting holes organize themselves, without anyone telling them to, into Thiessen polygons.

#### Does it work?

Well, here's a sample analysis.



This is not an easy choice of honeycomb holes because the holes are obviously of different sizes which complicates matters, but let's ignore that.

First, identify the holes:



Then draw the Thisessen diagram for these holes:



Then compare it with the actual real-life array of honeycomb holes:



I think the match is pretty good. There are obvious problems with the long slots on the righthand side, but if you regard these for what they are, namely an amalgamation of holes, the "problems" don't amount to much.

Here's another example from the cliffs on Valdes Island. In case you are wondering, the white "globes" near the bottom on the right are concretions.





The next picture is a selected portion of this marvellous honeycombing. I've reversed black and white in the photograph to give emphasis to the walls by making them black. On the right of the photograph is the Thiessen-polygon analysis. The letters A, B, and C are there simply to help you match the photograph with the diagram. They have no other significance.



## Conclusion

There is very little about the geometry of honeycomb holes that cannot be explained by the simple idea that they are created by salt left behind by water evaporating in the sun and drawn to the surface of the rock from deep inside. ◊

## Appendix 1—The depth of honeycomb holes

Although honeycomb holes deepen themselves by attracting moisture on its way from the interior of the rock to the surface and evaporating it on its floor leaving salt behind, there is a limit to the effectiveness of this process. The deeper the hole gets, the more its floor is shaded from the sun by its walls. Hence, as the hole gets deeper, the less efficient its floor becomes as an evaporation surface. This limits its depth.

To check this out, I calculated the illumination by the sun at the bottom of a hole. I actually did the calculation for a square hole to make the algebra a whole lot easier, but I'm sure the roundness of the hole makes little difference for present purposes. The loss of illumination of the floor of a hole has two components.

One is the daily component as the sun moves east to west. The floor is first shaded by the east side of the hole and then, after mid-day, the west side of the hole. Shading first thing in the morning and shortly before sunset is of less importance than shading during the day because at those times, the illumination is oblique and hence less effective anyway. I took this into account.

The second component is the annual component as the sun changes its altitude at noon by  $\pm 23.4^{\circ}$  from mid-summer to mid-winter and back. To simplify the calculation, I only considered the shading of a hole whose axis pointed south.

The other variable to be included is the inclination of the floor of the hole. For holes on vertical cliffs (90°) shading by the upper wall occurs throughout the year, and for holes that are horizontal ( $\approx$ 0°), shading by the lower wall—the one closest to south—also occurs throughout the year.

The happy medium, when shading by the upper and lower walls is equal, is when the axis of the hole points at the position of the sun at noon on the day of the equinoxes. The optimal inclination of the floor of the hole to the horizontal is thus its latitude, which for convenience I just took to be 50°.

The results of the calculation shown *left*, indicate that very little illumination occurs if the depth to diameter ratio is greater than

two. It isn't possible to say what the limiting ratio might be, but observation shows that it is around one, and this makes perfect sense. By then, shading has reduced the illumination to 27% or less.

Once shading of the floor is significant, the walls erode instead and the hole's diameter increases, which in turn reduces the shading of the floor, which in turn allows the hole to grow deeper. ◊





### Appendix 2—The shape of honeycomb holes





The floors of holes facing south are illuminated by the sun equally on the right and left (west and east) during the course of a day; however, the floors of holes on faces inclined more than 41° (90°– latitude) to the horizontal will on average be illuminated more on their lower side than their upper side during the course of a year because of shading by the upper wall.

The asymmetry in illumination, and hence evaporation and salt deposition (visible in the picture *left*), accounts in part for the asymmetry of some of the deeper holes and cavities. The effect may be stronger in the tropics where to catch the most sunlight, steep honeycombed cliffs face east or west rather than south, and the variation in sunlight happens in the course of a day rather than a year. Some holes in the picture *above* are examples of honeycombs within honeycombs.









Although honeycombing is common near the sea, it also occurs where rainfall is so low that salt is not regularly leached from the soil.

Picture *top left* is of one of the tombs at Mada'in Saleh (Hegra), Saudi Arabia, cut into a sandstone outcrop. The honeycombing of the outcrop has several characteristics of that seen in the Gulf Islands—large holes and denuded faces at the top *picture above*, and holes along bedding planes at the bottom *picture middle left*. This desert site is 150 km from the sea.

The elongation of the smaller holes along their vertical axes into slots in Saudi Arabia is more noticeable than it is in the Gulf Islands and may be a result of shading by the upper walls as the sun passes directly overhead. We saw possibly the same effect in tropical Australia (Kennedy Range, WA) on honeycombed sandstone slopes facing east, *picture lower left*. This site is also about 150 km from the sea. The subflorescence of salt causing the honeycombing was, in places, plainly visible.



The power of the sun. Had this photograph been taken on Gabriola, it would have been a fair bet that this honeycombed rock faced between southeast and southwest. But it wasn't, and it doesn't. It's near Manly, New South Wales, Australia, and it faces between northeast and northwest as do many other honeycombed surfaces in that country.

## Appendix 3—The orientations of honeycomb holes

The more sunlight a sandstone face receives the more likely it is to be honeycombed. Radiant heat is efficient at causing salt to crystallize just *below* the surface in the pores of the rock where it can do most damage rather than at the surface where it is relatively harmless (subflorescence rather than efflorescence). Given this, the question then becomes, for maximum honeycombing, what is the optimum inclination of the face, and what is the optimum azimuth? Both depend on latitude. These are of course the same questions faced by people installing solar panels for heating or electricity generation.

To get a feel for how critical the geometry of the cliff faces is, I computed the total sunlight received by a rock face during the course of a year at various inclinations and latitudes and as a function of which way it faced. The results are shown below. Although I allowed for the extra absorption of sunlight by the atmosphere when the sun is low in the sky, I only considered seasonal cloudiness in the last of the four diagrams.



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The results show that as expected facing south (azimuth 180<sup>o</sup>) is mostly beneficial—with apologies to Australian readers who have to do everything the other way round.

*Top left*: In the tropics (24°N), inclination to the horizontal makes most difference. Note that for vertical surfaces (inclination 90°), most radiation is not received facing south because at noon the sun is more or less directly overhead.

*Top right*: In polar regions (66<sup>o</sup>N), the value of the inclination to the horizontal makes relatively little difference for surfaces facing roughly south.

*Bottom left*: In temperate regions ( $45^{\circ}$ N) like Gabriola, the best inclinations are between  $30^{\circ}$  and  $60^{\circ}$ , but the exact value is not critical.

*Bottom right*: Note the scale change. In temperate regions  $(45^{\circ}N)$  with the effect of winter cloudiness included, the best inclinations are again between 30° and 60°. For vertical surfaces (inclination 90°), the exact value of azimuth is also not important. Facing anywhere between 110° (just south of east) and 250° (just south of west) produces the same result.

Evidently, the statement "the most honeycombed surfaces face south" is more accurately rendered as "the least honeycombed surfaces face north".  $\diamond$ 







No need to look for moss on a tree or ask a Boy Scout if you get lost in Descanso Bay. The honeycombs all face south (*top left*) leaving few facing north (*bottom left*), something you can see even when it's raining. *Above*: On north shore beaches (that's Entrance Island), honeycombs have a problem getting an unobstructed view of the sun, so they are confined to small undulations and ridges on the beach facing away from the sea.

## Appendix 4—Evaporation rates

One objection to the idea that honeycombing is driven by the greater evaporation rate on the floors of cavities than on the surrounding surfaces is that conditions within honeycomb cavities are unlikely to be favourable for evaporation. Air circulation will be poor, and humidity will be higher than outside the cavity. While this objection is to be taken seriously, I think it is wrong for the following reasons.

The rate at which water migrates toward the surface and evaporates leaving salt to accumulate is dependent on two groups of factors.

The first group encompasses the properties of the rock: how fast water moves toward the surface driven by differential capillary pressure.<sup>12</sup> This group includes the permeability of the rock, its porosity, and its pore geometry. It also includes the effect of osmosis. Water moves toward the surface to counteract the rising concentration of salt.

The second group encompasses the properties of the environment at the surface: how fast free-standing water evaporates. This second group includes the temperature at the surface, the relative humidity, and the movement of the air due to convection and wind. Estimating the effect of these two groups is best done using empirical data because so many factors are involved.

Work by Weisbrod *et al.*<sup>13</sup> gives us some experimental data for the ability of the rock

to deliver pore water to the evaporation surface. They used sandstone samples with a permeability of 0.1 darcy, or, in SI units, a hydraulic conductivity (K) of  $10^{-6}$  m/s at  $22^{\circ}$ C.<sup>14</sup> This is high for unfractured sandstone. Their second set of samples was of chalk with a permeability of 0.001 darcy, or, in SI units, a hydraulic conductivity of  $10^{-8}$  m/s at 22°C, which is within the normal range for sandstone.

The authors of the study identified three mechanisms for evaporation, namely, diffusion, water vapour transfer by thermal convection currents, and water vapour transfer by surface winds. It was envisaged that diffusion and some convection would always be present due to differences in temperature and moisture content between the air in the cavity and the air outside.<sup>15</sup> The "outside air" was kept at 21°C and RH 32 % for the measurements.

What was found was that, using 5 cm diameter surfaces and ignoring the initial period of 20 days when evaporation was high, the diffusion mechanism leads to an evaporation rate of 0.3–0.5 mL/day  $[1.8\times10^{-9} - 2.9\times10^{-9} \text{ m/s}]$ , and the convection mechanism with no forced air movement, between 5–12 mL/day  $[2.9\times10^{-8} - 7.1\times10^{-8} \text{ m/s}]$  for the sandstone, and  $\approx 4$ mL/day  $[2.4\times10^{-8} \text{ m/s}]$  for the chalk. We

<sup>&</sup>lt;sup>12</sup> Experiments with different kinds of sodium salts have show that the lower the surface tension (the larger the anion), the less is the weathering. Doe N., *Salt-weathering..., ibid*, p.56, March 2010.

<sup>&</sup>lt;sup>13</sup> Noam Weisbrod, Modi Pillersdorf, Maria Dragila, Chris Graham, James Cassidy, Clay Cooper, *Evaporation from fractures exposed at the land* 

surface—Impact of gas phase convection on salt accumulation, pp.151–164, in Boris Faybishenko, Paul Witherspoon, JohnGale (ed.), *Dynamics of fluids* in fractured rock, American Geophysical Union Monograph 162, 2005.

<sup>&</sup>lt;sup>14</sup> The conductivity K can be equated with the "drainage" velocity of the rock—(cubic metres/sec) per square metre. The concentration of salt is assumed to be low enough not to affect the calculation.

<sup>&</sup>lt;sup>15</sup> Relative temperatures however will fluctuate, possibly it being cooler in the shaded parts of the cavity than outside during the day, but the reverse during the night.

need to compare these figures with the rate of evaporation of water.

The interest in calculating the loss of water from aquariums and swimming pools gives many formulas for calculating evaporation losses. I'll use an empirical formula from the online engineering toolbox.<sup>16</sup> At 21°C and RH 32 %, with no wind, the evaporation rate from the surface of standing water is  $0.27 \text{ kg/m}^2/\text{h} [7.4 \times 10^{-8} \text{ m/s}].$ 

We now have the problem of deciding what proportion of the sandstone surface is an active evaporation surface linked by microfractures to the interior of the rock. I have to guess, and I'm assuming that it is about 5% based on the fact that the porosity of sandstone (unfractured) is 1–10%, and my own measurements show that wet Nanaimo Group sandstone loses 2.3% of its weight when dried out, making the volume of retained water 5.3% . So the evaporation rate from the rock surface is of the order of  $3.8 \times 10^{-9}$  m/s, or roughly the same magnitude as the rate observed for diffusion to an exposed surface.

It is therefore a fair bet that the rate at which evaporation takes place in a honeycomb cavity is dictated by the cavity environment and is not limited in some way by the rate at which the water is able to diffuse through the rock. This was assumed to be the case in making the spreadsheet calculations in the body of this article.

#### Measurements

To check the estimates, I measured the surface temperature of the floors of cavities and the surface temperature of the sandstone just outside these cavities using an Omega OS418L non-contact infrared thermometer. This instrument is also equipped with a relative humidity sensor. The humidity sensor has to be inserted into the cavity to obtain a reading; it is not a part of the infrared measuring system.

Two sets of measurements were made; one in the afternoon on a sunny day, and the other in the evening after sunset. All of the afternoon measurements inside the cavities were made shielded from direct sunlight. Results were as follows:

Afternoon (26 measurements): Air pressure: 101.2 kPa Cavity: surface temperature: 17.7°C rel. humidity: 42.6% H<sub>2</sub>O evap. rate:  $2.4 \times 10^{-9}$  m/s (5% porosity) Outside: surface temperature: 22.0°C rel. humidity: 41.9% H<sub>2</sub>O evap. rate:  $3.2 \times 10^{-9}$  m/s (5% porosity) Evening (28 measurements): Air pressure: 100.8 kPa Cavity: surface temperature: 16.0°C rel. humidity: 51.2% H<sub>2</sub>O evap. rate:  $1.8 \times 10^{-9}$  m/s (5% porosity) Outside: surface temperature: 14.6°C rel. humidity: 51.4% H<sub>2</sub>O evap. rate:  $1.65 \times 10^{-9}$  m/s (5% porosity) Evaporation was lower in the cavity than outside in the afternoon, but only by 25%. In the evening, evaporation was greater within the cavity than outside because of the retained warmth of the rock. Honeycomb cavities evidently can be favourable sites for evaporation, and I have no doubt that they often are.  $\diamond$ 

<sup>&</sup>lt;sup>16</sup> Evaporation from water surfaces,

http://www.engineeringtoolbox.com.

## Appendix 5—Biological honeycombing

Readers of the earlier paper on honeycombing will have seen an illustration of honeycomb patterns in the lung of a bird.<sup>17</sup> Let me now ask if you can spot the difference between the following two photographs?



Well, the one on the *right* is *tafoni* in sandstone. The one on the *left* is a photograph of pits that are roughly a million times smaller—the scale bar *bottom left* of the photo is a mere 5  $\mu$ m.<sup>18</sup> These pits are on the surface of a shell of the fossil of an extinct brachiopod, *Lingulella antiquissima*. Brachiopods are marine creatures similar to, but different from, bivalve molluscs like oysters, clams, and mussels.

The *Lingulata* class of brachiopods (Lingulids) happen to be of special interest to me because their fossils are common in the shale of the Northumberland Formation on Gabriola Island. They are however in poor shape, and their chances of making it to an off-island museum are near zero, but one characteristic they have that makes recognition easier is that their shells were (and are—some species still exist) organophosphatic; that is they contain calcium phosphate (*apatite*), not as other brachiopods and all bivalves do, calcium carbonate (*calcite*).



The explanation for the pattern is truly complicated for non-specialists like myself, but my over-simplified version goes like this. In order to build a shell, an organism has to create in its epidermis (skin) micro-spaces isolated from the environment within which sufficient ions for crystallization can be accumulated. This protective enclosure is provided by a thin outermost organic layer called a *periostracum*, which is often lost once shell construction is complete. Mineral deposition occurs between the periostracum and underlying mantle. It is surmised by the researchers, that the pits in the shell are casts of bodies, possibly membrane-bound mucus, secreted by a basal epithelial layer to construct a framework to be subsequently infilled with phosphatized material. These bodies would have initially been spheroidal but deformable, so they shaped themselves before polymerization to accomodate neighbours and leave inter-pit borders as they grew. Their geometry evidently has a lot in common with that of honeycombs in sandstone.  $\Diamond$ 

<sup>&</sup>lt;sup>17</sup> Doe, N.A., *What makes holes in sandstone*, *SHALE* 9, pp.19–20, August 2004.

<sup>&</sup>lt;sup>18</sup> Plate 1 (7), from Cusack, M., Williams A., & Buckman, J.O., *Chemico-structural evolution of linguloid brachiopod shells*, Paleontology, 42, 5, pp.799–840, 1999.